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Inventory Management Model with Quadratic Demand, Variable Holding Cost with Salvage value

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Abstract

In this paper inventory management model for deteriorating products with quadratic function of time as rate of demand. Here the deterioration is considered as weibull deterioration rate. Shortages are allowed. The salvage value is also used for deteriorated items in the system. The total cost is calculated by considering variable holding cost. Suitable values for the parameters considered in the numerical example. Sensitivity analysis is also discussed.

Keywords: Quadratic demand, weibull deterioration rate, salvage value, shortages.

Introduction

Researchers had been developed inventory models by assuming constant demand rate for the items like electronic goods, vegetables, food stuffs, fashionable clothes etc., since the demand rate is always fluctuating and introducing new products will attract more in demand. Due to customer's choice and arrival of new product in the markets, normally some product may decline in demand rate. Attempting the phenomenon of time-varying demand pattern in the deteriorating inventory models yields very much real time application. So far in developing Inventory models, there are two kinds of time-varying demands namely discrete time and Continuous time. Many of the continuous –time inventory models were developed by considering like linearly increasing /decreasing demand patterns. Ghare and Schrader studied an inventory management model incorporating that the rate of demand is decaying exponentially[1]. Covert and Philip studied an inventory model with deterioration rate is time dependent[2]. Aggarwal discussed an inventory model for a system with rate of deterioration is constant[3]. Dave and Patel proposed an EOQ model for time proportional demand with constant deterioration[4]. Deb and Choudhuri studied an inventory model considering trended inventories by assuming shortage[5]. Hariga discussed an inventory model for time-varying demand of deteriorating products by considering shortages are allowed[6]. Chakraborti and Choudhuri proposed an inventory model for deteriorating items of linear demand also in all cycles shortages are allowed[7]. Giri and Chaudhuri studied a heuristic model for deteriorating items of time varying demand and costs with shortages[8]. Goyal and Giri thoroughly studied survey of recent trend in deteriorating inventory models[9]. Mondal *et. al* discussed EOQ model for ameliorating products by considering price dependent demand[10]. You proposed an inventory system for the items with time and price dependent demands[11].

Ajanta Roy proposed an EOQ model with demand rate is price dependent and incorporating variable holding cost with respect to time by considering with / without shortages of deteriorating products[12]. Mishra and Singh studied an inventory model time dependent demand of deteriorating items considering partial backlogging[13]. Mishra proposed an inventory model of constant demand with Weibull rate of deterioration. He incorporated variable holding cost considering shortages and salvage value[14]. Vikas Sharma and Rekha studied an EOQ model for time dependant demand for deteriorating products with Weibull deterioration rate, also considering shortages[15].

Venkateswarlu and Mohan proposed an EOQ model with 2-parameter Weibull deterioration, time dependent quadratic demand and salvage value[16]. Venkateswarlu and Mohan developed an EOQ model for time varying deterioration and price dependent quadratic demand with salvage value[17]. Mohan and Venkateswarlu studied an inventory management models with variable holding cost and salvage value[18]. Mohan and Venkateswarlu proposed an inventory model for, time Dependent quadratic demand with salvage considering deterioration rate is time dependent[19].

In this paper, we have considered an order level inventory problem when the demand rate is a quadratic function of time with Weibull deterioration and variable holding Cost is incorporated. Shortages are allowed. The time horizon is infinite. Salvage value also considered for the optimal total cost. Suitable numerical example and sensitivity analysis is also done.

Assumptions and notations

We incorporated following notations and assumptions to develop mathematical model: The rate of demand $D(t)$ at any time t is assumed to be $D(t) = at^2+bt+c$, $a \geq 0, b \neq 0, c \neq 0$, Rate of replenishment is infinite , Lead time is zero, A, the ordering cost per order , $\theta(t) = \alpha\beta t^{\beta-1}$, 2-Parameter Weibull Deterioration rate, $0 < \theta < 1$, C, the cost per unit per order, $h+rt$, $h>0$, $r>0$, the holding cost per unit, $I(t)$ is the inventory level at time t., q, is the order quantity in one cycle, γ*C, $0 \leq \gamma < 1$, the salvage value associated with deteriorated units during a cycle time, NDU, the number of deteriorating units per order with one cycle time, π , the cost of shortages per unit per order

Formulation and solution of the model

The governing differential equation which describes variation of inventory w.r.to to't' (time) is

$$\frac{d(Q_1(t))}{dt}+\theta(t)Q_1(t)=-(at^2+bt+c);\theta=\alpha\beta t^{\beta-1} \quad 0\leq t\leq t_1 \qquad (1)$$

$$\frac{d(Q_2(t))}{dt}=-(at^2+bt+c); \quad t_1\leq t\leq T \qquad (2)$$

with $Q_1(t) = Q_2(t) = 0$ at $t = t_1$.

Equation (1) is a linear first order equation, hence

$$\left(Q_1(t)e^{\alpha t^\beta}\right)' = -(at^2+bt+c)e^{\alpha t^\beta}$$

On integration, the above equation yields

$$Q_1(t)e^{\alpha t^\beta} = -\left(\frac{at^3}{3}+\frac{bt^2}{2}+ct\right)-\alpha\left(\frac{at^{\beta+3}}{\beta+3}+\frac{bt^{\beta+2}}{\beta+2}+\frac{ct^{\beta+1}}{\beta+1}\right)+k_1$$

where k_1 is an integral constant. Here we have expanded $e^{\alpha t^\beta}$ and ignored higher order terms as α is small.

Using the given boundary conditions, solution of the above differential equation is given by

$$Q_1(t)=\left\{\begin{array}{l}\frac{a}{3}(t_1^3-t^3)+\frac{b}{2}(t_1^2-t^2)+c(t_1-t)+\\ \alpha\left[\begin{array}{l}\frac{a}{\beta+3}(t_1^{\beta+3}-t^{\beta+3})+\frac{b}{\beta+2}(t_1^{\beta+2}-t^{\beta+2})\\ +\frac{c}{\beta+1}(t_1^{\beta+1}-t^{\beta+1})\end{array}\right]\end{array}\right\}e^{-\alpha t^\beta} \qquad (3)$$

$$Q_1(t)=\left\{\begin{array}{l}\frac{a}{3}(t_1^3-t^3)+\frac{b}{2}(t_1^2-t^2)+c(t_1-t)+\\ \alpha\left[\begin{array}{l}\frac{a}{\beta+3}(t_1^{\beta+3}-t^{\beta+3})+\frac{b}{\beta+2}(t_1^{\beta+2}-t^{\beta+2})\\ +\frac{c}{\beta+1}(t_1^{\beta+1}-t^{\beta+1})\end{array}\right]\end{array}\right\}(1-\alpha t^\beta)$$

Since $Q_1(0) = Q$, we get

$$Q_1(t)=\left(\frac{at_1^3}{3}+\frac{bt_1^2}{2}+ct_1\right)+\alpha\left[\frac{at_1^{\beta+3}}{\beta+3}+\frac{bt_1^{\beta+2}}{\beta+2}+\frac{ct_1^{\beta+1}}{\beta+1}\right] \qquad (4)$$

Solving equation (2) and its solution is given by

$$Q_2(t) = \left[\frac{a}{3}(t_1^3-t^3)+\frac{b}{2}(t_1^2-t^2)+c(t_1-t)\right]$$

In the cycle time the number of deteriorating units (NDU) is given by

$$\text{NDU} = Q - \int_0^{t_1} D(t)dt \qquad (6)$$

where D(t) = (at^2+bt+c) is the rate of Demand.

The number of deteriorated units (NDU)

$$=\left(\frac{at_1^3}{3}+\frac{bt_1^2}{2}+ct_1\right)+\alpha\left[\frac{at_1^{\beta+3}}{\beta+3}+\frac{bt_1^{\beta+2}}{\beta+2}+\frac{ct_1^{\beta+1}}{\beta+1}\right]-\left(\frac{at_1^3}{3}+\frac{t_1^2}{2}+ct_1\right)$$

$$=\alpha\left[\frac{at_1^{\beta+3}}{\beta+3}+\frac{bt_1^{\beta+2}}{\beta+2}+\frac{ct_1^{\beta+1}}{\beta+1}\right]$$

Cost due to deterioration (CD) =

$$C\alpha\left[\frac{at_1^{\beta+3}}{\beta+3}+\frac{bt_1^{\beta+2}}{\beta+2}+\frac{ct_1^{\beta+1}}{\beta+1}\right] \qquad (7)$$

$$\text{Salvage value (SV)} = \gamma C\alpha\left[\frac{at_1^{\beta+3}}{\beta+3}+\frac{bt_1^{\beta+2}}{\beta+2}+\frac{ct_1^{\beta+1}}{\beta+1}\right] \qquad (8)$$

Inventory time varying holding cost (IHC) in the interval (0, t1) is

$$IHC = \int_{o}^{t_1} Q_1(t)(h+rt)dt$$

$$= \int_{0}^{t_1} \left\{ \begin{array}{l} \frac{a}{3}(t_1^3 - t^3) + \frac{b}{2}(t_1^2 - t^2) + c(t_1 - t) + \\ \alpha \left[\frac{a}{\beta+3}(t_1^{\beta+3} - t^{\beta+3}) + \frac{b}{\beta+2}(t_1^{\beta+2} - t^{\beta+2}) \right. \\ \left. + \frac{c}{\beta+1}(t_1^{\beta+1} - t^{\beta+1}) \right] \end{array} \right\} (1-\alpha t^{\beta})(h+rt)dt$$

$$= \int_{0}^{t_1} hQ_1(t)dt + \int_{0}^{t_1} rtQ_1(t)dt$$

$$= h\left[\frac{a}{4}(t_1^4) + \frac{b}{3}(t_1^3) + c\left(\frac{t_1^2}{2}\right) + \left(\frac{\alpha\beta}{\beta+1}\right)\left[a\left(\frac{t_1^{\beta+4}}{\beta+4}\right) + b\left(\frac{t_1^{\beta+3}}{\beta+3}\right) + c\left(\frac{t_1^{\beta+2}}{\beta+2}\right) \right] \right]$$

IHC

$$+ r\left[\frac{at_1^5}{10} + \frac{bt_1^4}{8} + \frac{ct_1^3}{6} + \left(\frac{\alpha}{2(\beta+2)}\right)\left[\frac{t_1^{\beta+5}}{(\beta+5)} + \frac{bt_1^{\beta+4}}{(\beta+4)} + \frac{ct_1^{\beta+3}}{(\beta+3)} \right] \right] \quad (9)$$

Ordering Cost = A Shortage cost = $\int_{t_1}^{T} -\pi Q_2(t)dt$

$$= -\pi\left\{ \frac{a}{3}\left(t_1^3 T - \frac{T^4}{4} - \frac{3t_1^4}{4}\right) + \frac{b}{2}\left(t_1^2 T - \frac{T^3}{3} - \frac{2t_1^3}{3}\right) + c\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) \right\} \quad (10)$$

Total cost (TC) of the system is given by
TC = (OC+IHC+SC+CD-SV)

$$= \frac{1}{T} \left\{ \begin{array}{l} A + h\left[\frac{a}{4}(t_1^4) + \frac{b}{3}(t_1^3) + c\left(\frac{t_1^2}{2}\right) + \left(\frac{\alpha\beta}{\beta+1}\right)\left[a\left(\frac{t_1^{\beta+4}}{\beta+4}\right) + b\left(\frac{t_1^{\beta+3}}{\beta+3}\right) + c\left(\frac{t_1^{\beta+2}}{\beta+2}\right) \right] \right] \\ + r\left[\frac{at_1^5}{10} + \frac{bt_1^4}{8} + \frac{ct_1^3}{6} + \left(\frac{\alpha}{2(\beta+2)}\right)\left[\frac{at_1^{\beta+5}}{(\beta+5)} + \frac{bt_1^{\beta+4}}{(\beta+4)} + \frac{ct_1^{\beta+3}}{(\beta+3)} \right] \right] \\ - \pi\left\{ \frac{a}{3}\left(t_1^3 T - \frac{T^4}{4} - \frac{3t_1^4}{4}\right) + \frac{b}{2}\left(t_1^2 T - \frac{T^3}{3} - \frac{2t_1^3}{3}\right) + c\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) \right\} \\ + (1-\gamma)C\alpha\left[\frac{at_1^{\beta+3}}{\beta+3} + \frac{bt_1^{\beta+2}}{\beta+2} + \frac{ct_1^{\beta+1}}{\beta+1} \right] \end{array} \right\} \quad (11)$$

The necessary conditions for minimizing the total cost is

$$\frac{\partial(TC(t_1,T))}{\partial t_1} = 0, \quad \frac{\partial(TC(t_1,T))}{\partial T} = 0, \text{ and}$$

$$\left(\frac{\partial^2(TC(t_1,T))}{\partial t_1^2}\right)\left(\frac{\partial^2(TC(t_1,T))}{\partial T^2}\right) - \left(\frac{\partial^2(TC(t_1,T))}{\partial t_1 \partial T}\right)^2 > 0$$

Numerical Example

By putting proper units of parametric values for
A =100 a = 10, b = 25, c = 100, C = 4, β = 0.9, h = 0.6, r = 0.3, α = 0.2, π= 10, γ = 0.2

Using MATHCAD Software, the optimal values the inventory system are
t_1= 0.902, T = 1.032, q = 125.228, TC = 174.116.

Sensitivity Analysis

From this model we analyze the robustness of parameter values A, π ,a, b, α, C, and γ on the optimal cycle time, TC (total cost) and EOQ of this model. From table-1 the inferences are as follows: i. Ordering quantity (Q) and the cycle time (T) increases (decreases) whereas and the total cost (TC) increase (decrease) with the increase (decrease) in the values of ‘a’ but the rate of change is insignificant. ii. We notice that the changes in the values of T (cycle time), the ordering quantity (Q) and the total system cost are similar when the parameters *b* and *c* are over estimating or underestimating. However the parameter *c* has distinct effect on all these values. iii. The effect of α on the cycle time and the ordering quantity is quite similar while it is different on the total system cost. But the rate of change on these values is almost same. iv. The effect of the parameters *A* and *C* on the optimum total cost value is similar but the changing rate is significant in case of *A*. The cycle time and ordering quantity increase (decrease) with an increase (decrease) in case of A_0 but the effect is quite opposite in case of *C*. v. The effect of the parameters γ and π on the optimum cycle time, ordering quantity and total cost is quite opposite when we increase or decrease the values of these two parameters. vi. The total cost is highly sensitive than the T and ordering quantity when the values of all parameters are under-estimated or over-estimated by 50%.

Conclusion

We have developed an EOQ model for deteriorating products by assuming rate of demanded is quadratic with respect to time. In this paper, the deterioration rate follows 2-parameter Weibull distribution. We have solved this model with variable holding cost and shortages. Analyzing this model the TC (total cost) is high sensitivity than the cycle time and ordering quantity when the values of all parameters are overestimated or underestimated.

Table-1
Sensitivity analysis for the parameters A, π, a, b, α, C, and γ -50% to +50%

Parameter	% change	t_1	T	q	TC
A	-50	-24.3902	-24.612403	-29.1117	-31.5899
	-20	-8.42572	-8.5271318	-10.555147	-11.6175
	+20	7.317073	7.46124031	9.68633213	10.72446
	+50	16.85144	17.1511628	22.9716996	25.5818
a	-50	1.995565	2.03488372	0.89596576	-0.89193
	-20	0.776053	0.7751938	0.33299262	-0.35034
	+20	-0.77605	-0.6782946	-0.248347	0.342875
	+50	-1.77384	-1.744186	-0.6947328	0.842542
b	-50	4.767184	4.84496124	-0.1860606	-3.11344
	-20	1.773836	1.84108527	-0.0758616	-1.20954
	+20	-1.66297	-1.6472868	0.16449995	1.167612
	+50	-3.88027	-3.9728682	0.37771106	2.848101
c	-50	16.07539	16.375969	-28.526368	-19.8236
	-20	5.764967	5.91085271	-10.506436	-7.55761
	+20	-4.98891	-5.0387597	9.79653113	7.141216
	+50	-11.5299	-11.531008	23.1401923	17.18797
α	-50	12.63858	9.39922481	9.82607724	-10.5688
	-20	4.545455	3.39147287	3.59743827	-4.01399
	+20	-4.102	-2.9069767	-3.1310889	3.772198
	+50	-9.53437	-6.7829457	-7.3705561	9.036504
C	-50	10.86475	7.94573643	10.9065065	-9.80611
	-20	3.991131	3.00387597	4.01427796	-3.75784
	+20	-3.65854	-2.6162791	-3.4409238	3.564865
	+50	-8.64745	-6.2015504	-8.0309515	8.585656
γ	-50	-2.32816	-1.6472868	-2.1792251	2.24965
	-20	-0.99778	-0.6782946	-0.9071454	0.90859
	+20	0.997783	0.7751938	1.02133708	-0.92065
	+50	2.439024	1.84108527	2.44753569	-2.32489
π	-50	-5.43237	6.39534884	6.5288913	-5.70252
	-20	-1.44124	1.64728682	1.64979078	-1.52887
	+20	0.997783	-1.0658915	-1.047689	1.063084
	+50	1.995565	-2.2286822	-2.195196	2.162352
All parameters	-50	14.30155	19.2829457	-39.158974	-59.465
	-20	6.097561	6.97674419	-13.670265	-26.2354
	+20	-5.87583	-6.1046512	11.6667199	29.49413
	+50	-13.5255	-13.953488	26.3215894	79.3919

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