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An inventory Model for deteriorating items with Weibull Deterioration with Time Dependent Demand and Shortages

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Abstract

This paper deals with in developing an inventory model for deteriorating items, the rate of deterioration follow the Weibull distribution with two parameters. The demand rate is assumed of time dependent. The shortages are allowed and shortages are completely backlogged. The numerical example is given to illustrate the model developed. The model is solved analytically by maximizing the total profit.

Keywords: Inventory, EOQ Model, deteriorating items, Weibull distribution of two parameters, shortages, linear demand dependent of time.

Introduction

Deterioration of physical goods is one of the important factors in any inventory and production system. The deteriorating items with shortages have received much attention of several researches in the recent year because most of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables and food stuffs from depletion by direct spoilage wile kept in store.

Ghare and Schrader[1] developed a model for an exponentially decaying inventory. An order level inventory model for items deteriorating at a constant rate was proposed by Shah and Jaiswal[2], Aggarwal[3], Dave and Patel[4]. Inventory models with a time dependent rate of deterioration were considered by Covert and Philip [5], Mishra[6] and Deb and Chaudhuri[7]. Some of the significant recent work in this field have been done by Chung and Ting[8], Fujiwara[9], Hariga and Benkherouf[10], Wee[11], Jalan et al.[12],Chakraborty and chaudhuri[13], Giri and Chaudhuri[14], Chakraborty, et al.[15] and Jalan and Chaudhuri[16], Structural properties of an inventory system with deterioration and trended demand. Burwell[17] developed economic lot size model for price-dependent demand under quantity and freight discounts. Inventory model for ameliorating items for price dependent demand rate was proposed by Mondal et.al.[18] and inventory model with price and time dependent demand was developed by you[19].

In general holding cost is assumed to be known and constant. But in realistic condition holding cost may not Goh[20] considered various functions to describe holding cost. In this paper we developed an EOQ Model for deteriorating items with deterioration rate, where deterioration rate follows two-parameters Weibull distribution and demand is consider as linear function of time. In this model shortages are completely back logged.

Assumptions and Notations

The fundamental assumptions are used to develop the model. The demand rate is dependent on time t.
D(t) = (a+bt) or b(t)
The deterioration rate is proportional to time. The cycle length is T. The ordering quantity is q. The ordering cost is A. The selling price per unit item is s. The deterioration cost per unit item per unit time cost C_2. and the deterioration rate is proportional to time.

The inventory holding cost per unit item per unit time is h. C_1 is the shortage cost per unit item per unit time.

The deterioration of time as follows by Weibull parameter (two) distribution $\theta(t) = \alpha\beta t^{(\beta-1)}$

Where $0<\alpha< 1$ is the scale parameter and $\beta >0$ is the shape parameter.

Formulation and Solution

The length of the cycle is T. during time t_1 inventory is depleted due to deterioration and demand of the items. At the time t_1 the inventor level becomes zero and shortages occurring in the period (t_1, T) which is completely backlogged. Let I(t) be the inventory level at time t ($0\leq t < T$).

The differential equation Can be defined when the instantaneous state over (O,T) are given by

$$\frac{dI(t)}{dt} + \alpha\beta(t)^{\beta-1} I(t) = -bt \quad O \leq t \leq t_1 \tag{1}$$

$$\frac{dI(t)}{dt} = -b(t) \qquad t_1 \leq t \leq T \tag{2}$$

With $I(t_1) = 0$ at $t = t_1$

From equation (1) we get

I (t) = b (1-∝ t^{β}) $\left[\frac{1}{2}(t_1^2 - t^2) + \frac{\propto}{\beta+2}\left(t_1{}^{(\beta+2)} - t^{(\beta+2)}\right)\right]$ O < t ≤ t_1 (3)

From equation (2) we get

I (t) = $\frac{b}{2}\ [t_1^2 - t^2]$ t_1 ≤ t ≤ T (4)

The holding cost during the time period 0 to t_1

H = $\int_0^{t_1} I\ (t)dt$

The total holding cost during the time period 0 to t_1

H = $h.\int_0^{t_1} I\ (t)dt$

H = h $\int_0^{t1} b(1-\propto t^{\beta})\left[\frac{1}{2}(t_1^2 - t^2) + \frac{\propto}{\beta+2}\left(t_1{}^{(\beta+2)} - t^{(\beta+2)}\right)\right]dt$

Now holding cost will be

H=bh$\left[\frac{2t_1^3}{6} - \frac{\propto}{2(\beta+1)}t_1^{(\beta+3)} + \frac{\propto}{2}\frac{t_1^{(\beta+3)}}{\beta+3} - \frac{\propto t_1^{(\beta+3)}}{\beta+3} - \frac{\alpha^2 t_1^{(2\beta+3)}}{\beta+1} - \frac{\propto t_1^{(2\beta+3)}}{(\beta+2)(\beta+3)} + \frac{\propto t_1^{(2\beta+3)}}{(\beta+2)(2\beta+3)}\right]$

H = bh $\left[\frac{2t_1^3}{6} + \frac{\alpha t_1^{(\beta+3)}}{2(\beta+1)} - \frac{\propto t_1^{(\beta+3)}}{(\beta+2)}\left(1 + \frac{1}{(\beta+3)}\right) + \frac{\propto t_1^{(2\beta+3)}}{(\beta+2)(2\beta+3)} + \frac{\propto t_1^{(\beta+3)}}{2\ (\beta+3)} - \frac{\propto^2 t_1^{(2\beta+3)}}{(\beta+1)}\right]$

H = bh $\left[\frac{t_1^3}{3} - \frac{\alpha}{(\beta+1)(\beta+3)}t_1^{\beta+3} - \frac{\alpha}{(\beta+2)}\left[\frac{\beta+4}{\beta+3}\right]t_1^{\beta+3} + \frac{\alpha t_1^{(2\beta+3)}}{(\beta+2)(2\beta+3)} - \frac{\alpha^2 t_1^{(2\beta+3)}}{(\beta+1)}\right]$ (5)

The total Deterioration cost during the time period 0 to t_1 is given by

$D_T = C_2\int_0^{t_1}\theta(t).I(t)dt$

$D_T = C_2\int_0^{t_1}\alpha.\beta t^{(\beta-1)}(b(1-\propto t^{\beta})\left[\frac{1}{2}(t_1^2 - t^2) + \frac{\alpha}{(\beta+2)}[t_1^{(\beta+2)} - t^{(\beta+2)}]\right]dt$

$D_T = C_2\alpha\beta b\int_0^{t_1}\left(t^{\beta-1} - \alpha t^{\ (2\beta-1)}\right)\frac{1}{2}(t_1^2 - t^2) + \frac{\alpha}{(\beta+l)}t^{(\beta-1)}[t_1^{(\beta+2)} - t^{(\beta+2)}]dt$

$D_T = C_2\alpha\beta b\left[\frac{1}{2}\frac{t_1^{(\beta+2)}}{\beta} - \frac{\alpha}{4\beta}t_1^{(2\beta+2)} - \frac{t_1^{(\beta+2)}}{2(\beta+2)} + \frac{\alpha}{4}\frac{t_1^{2(\beta+1)}}{(\beta+1)} + \frac{\alpha}{(\beta+2)}\left(\frac{t_1^{2(\beta+1)}}{\beta}\right) - \frac{\alpha}{(\beta-2)}\left(\frac{t_1^{2(\beta+1)}}{\beta}\right)\right]$

$D_T = C_2\alpha\beta b\left[\frac{1}{2}\left(\frac{1}{\beta} - \frac{1}{(\beta+2)}\right)t_1^{(\beta+2)} - \left(\frac{\alpha}{4\beta} - \frac{\alpha}{4(\beta+1)} - \frac{\alpha}{(\beta+2)\beta}\right)t_1^{2(\beta+1)} - \frac{\alpha}{(\beta+2)}\left(\frac{t_1^{2\beta}}{2\beta}\right)\right]$

$D_T = C\alpha_2\beta b\left[\frac{1}{\beta(\beta+l)}t_1^{\beta+2} + \alpha\left(\frac{(3\beta+2)}{4\beta(\beta+1)(\beta+2)}\right)t_1^{2(\beta+1)} - \frac{\alpha}{(\beta+2)}\left(\frac{t_1^{2\beta}}{2\beta}\right)\right]$ (6)

Now the total shortage cost during the time period t_1 to T is given by

$Sh = -C_1\int_{t_1}^{T} I(t)dt$

$Sh = -C_1\int_{t_1}^{T}\frac{b}{2}(t_1^2 - t^2)\ dt$

$Sh = \frac{C_2}{2}b\ \left[\frac{T^3}{3} + \frac{2t_1^3}{3} - t_1^2 T\right]$ (7)

From Equation (5), (6) and (7) the total profit per unit time can define

P(T, t_1) = s.(bt) - $\frac{1}{T}[A + H + D_T + S_h]$

P (T, t_1) = s.(bt) - $\frac{1}{T}\left[A + hb\left\{\frac{t_1^3}{3} - \frac{\alpha}{(\beta+1)(\beta+3)}t_1^{(\beta+3)} - \frac{\alpha}{(\beta+2)}\left(\frac{\beta+4}{\beta+3}\right)t_1^{(\beta+3)} + \frac{\alpha t_1^{(2\beta+3)}}{(\beta+2)(2\beta+3)} - \frac{\alpha^2 t_1^{(2\beta+3)}}{(\beta+1)}\right\} + \frac{C_1}{2}b\left\{\frac{T^3}{3} + \frac{2t_1^3}{3} - t_1^2 T\right\} + \right]$ (8)

Our main objective to maximize the profit function P(T, t_1), the necessary condition for maximize the profit are $\frac{\partial P(T,t_1)}{\partial T}, 0\ and\ \frac{\partial P}{\partial t_1}(T, t_1) = 0$

$-\frac{1}{T^2}\left[A + hb\left\{\frac{t_1^3}{3} - \frac{\alpha}{(\beta+1)(\beta+3)}t_1^{(\beta+3)} - \frac{\alpha}{(\beta+2)}\left(\frac{\beta+4}{\beta+3}\right)t_1^{(\beta+3)} + \frac{\alpha t_1^{(2\beta+3)}}{(\beta+2)(2\beta+3)} - \frac{\alpha^2 t_1^{(2\beta+3)}}{(\beta+1)}\right\} + \frac{C_1}{2}b\left\{\frac{T^3}{3} + \frac{2t_1^3}{3} - t_1^2 T\right\} + \right.$

$\left.C\alpha_2\beta b\left\{\frac{1}{\beta(\beta+l)}t_1^{\beta+2} + \alpha\left(\frac{(3\beta+2)}{4\beta(\beta+1)(\beta+2)}\right)t_1^{2(\beta+1)} - \frac{\alpha}{(\beta+2)}\left(\frac{t_1^{2\beta}}{2\beta}\right)\right\}\right] = 0$ (9)

And

$-\frac{1}{T}\left[hb\left\{t_1^2 - \frac{\alpha}{(\beta+1)}t_1^{(\beta+2)} - \frac{\alpha}{(\beta+2)}(\beta+4)t_1^{(\beta+2)} + \frac{\alpha}{(\beta+2)}t_1^{2(\beta+2)} - \alpha^2\frac{(2\beta+3)}{(\beta+1)}t_1^{2(\beta+1)}\right\} + \frac{c_1 b}{2}\{2t_1^2 - 2t_1 T\} + c_2\alpha\beta b\left\{\frac{1}{\beta}t_1^{(\beta+1)} + \right.\right.$

$\left.\left.\alpha\frac{(3\beta+2)}{2\beta(\beta+2)}t_1^{(2\beta+1)} - \frac{\alpha}{(\beta+2)}t_1^{(2\beta+1)}\right\}\right] = 0$ (10)

Using the software mathematica-5.1, we can calculate the optimal value of T* and t_1* simultaneously by equation no. (8) and equation no. (9). and the optimal value P* (T, t_1) of the average net profit is determined by equation no. 7. The optimal value of T*and t_1*, satisfy the sufficient conditions for maximizing profit function p* (T,t_1) are.

$$\frac{\partial^2 P}{\partial T^2}(T, t_1) < 0 \qquad \frac{\partial^2 P}{\partial t_1^2}(T,t_1) < 0 \qquad (11)$$

And $\frac{\partial^2 P}{\partial T^2}(T, t_1). \frac{\partial^2 P(T,t_1)}{\partial t_1^2} - \frac{\partial^2 P}{\partial T \partial t}(T, t_1) > 0$ And at T = T* optimal value $t_1 = t_1$ (12)

Numerical example

Example - 1. Let us consider A = 300, b = 200, h = 1.3, c_1 = 1.1, $\alpha = 0.1, \beta = 0.3$

Based on above input data and Using the software mathematica-5.1, we calculate the optimal value of P (T, t_1), T* and t_1* simultaneously by equation no. (8), equation no. (9) And equation no. (10).

P (T, t_1) = 1282.61, T* = 2.75312, t_1* = 1.478321

Example -2. Let us consider A = 350, b = 175, h = 1.2, c_1 = 1.02, $\alpha = 0.15, \beta = 0.35$

P (T, t_1) = 1381.50, T* = 2.32141, t_1* = 1.37592

Conclusion

In this paper we have developed an inventory model for deteriorating items, such as fruits, vegetables and food stuffs from depletion by direct spoilage while kept in store. The rate of deterioration follows the Weibull distribution with two parameters. The demand rate is assumed of time dependent. The shortages are allowed and shortages are completely backlogged. The deterioration cost, inventory holding cost and, shortage cost are considered in this model. The numerical examples are given to illustrate the model developed. The model is solved analytically by maximizing the total profit. In the numerical examples we found the maximum value of profit.

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