Research Journal of Management Sciences ISSN 2319–1171
Vol. **2(11),** 1-15, November **(2013)** *Res. J. Management Sci.*

Modeling the Selection of Returns Distribution of G7 Countries

G.S. David Sam Jayakumar and Sulthan A.
Jamal Institute of Management, Tiruchirappalli, South India, INDIA

Available online at: www.isca.in, www.isca.me
Received 23rd August 2013, revised 9th September 2013, accepted 26th October 2013

Abstract

The purpose of the study is to identify the statistical distribution that is followed by the Indices returns of G7 countries. Canada is one of the members of the G7 countries but the data is insufficient and so it is not considered for the study. The closing values of indices were collected for selected countries from July 2003 to February 2013. The general assumption is that the stock returns are normally distributed. Using a statistical software 11 unbounded distributions was fitted for all the selected indices. The results show the returns follow different distributions that vary between countries.

Keywords: Unbounded distribution, G7 countries, Indices returns.

Introduction

The assumption that stock returns are normally distributed is widely used, implicitly or explicitly, in theoretical finance. The fitting of probability distributions to financial data is a statistical subject with a long tradition in both actuarial and financial literature. It was Louis Bachelier[1] who used a stochastic approach to model financial time series for the first time. In 1973, Fischer Black and Myron Scholes[2] published their famous work where they presented a model for pricing European options. They assumed that a price of an asset can be described by a geometric Brownian motion. However, Mandelbrot[3] showed the behaviour of real markets differs from the Brownian property, since the price returns form a truncated Levy distribution[4,5]. As a result of this observation many non-Gaussian models were introduced by Mantegna and Stanley[6] and Bouchaud[7]. Another divergence from the Gaussian behaviour is an autocorrelation in financial systems. Empirical studies show that the autocorrelation function of the stock market time series decays exponentially with a characteristic time of a few minutes, while the absolute values of the autocorrelation of prices decay more slowly, as a power law function, which leads to a volatility clustering[8, 9]. Jansen and De Vries[10], used extremes to investigate the fatness of the distribution tails. In this study, the authors fit 11 unbounded distributions to selected countries to understand the statistical distribution followed by the country's index returns.

Methodology

In case of time series analysis selecting an appropriate distribution is very important to get a meaning full result from the analysis however the nature of data differ from each other this necessary to find its nature of distribution so this study was made to find distribution G7 countries. The closing values of index for G7 countries from July 2003 to February 2013 was collected from yahoo finance website. After secondary data collection is over the returns were analyzed with help of math values easy fit 5.5.the analysis done in different stage in first stage in descriptive statistic identify for the returns of G7 countries. In next stage eleven unbounded distribution for fitted for all six countries. Canada is one of the members of the G7 countries but the data is insufficient and so it is not considered for the study.

Theoretical frame work of unbounded probability distribution: Gumbel min Distribution: In probability theory and statistics, the Gumbel distribution is used to model the distribution of the maximum (or the minimum) of a number of samples of various distributions. Such a distribution might be used to represent the distribution of the maximum level of a river in a particular year if there was a list of maximum values for the past ten years. It is useful in predicting the chance that an extreme earthquake, flood or other natural disaster will occur. The cumulative distribution function of the Gumbel distribution is $F(x;\mu,\beta) = e^{-e^{-(x-\mu)/\beta}}$

The mode is μ, while the median is $\mu - \beta\sum_{i=1}^{n} X_i \ln(\ln 2)$ and the mean is given by $E(X) = \mu + \gamma\beta$

Where γ =Euler-Mascheroni constant ≈ 0.5772

The standard deviation is $\beta\pi / \sqrt{6}$

Cauchy Distribution: The Cauchy distribution, named after Augustine, is a continuous probability distribution. It is also known, especially among physicists, as the Lorentz distribution (after Hendrik Lorentz), Cauchy–Lorentz distribution, Lorentz (ian) function, or Breit – Wigner distribution. The simplest Cauchy distribution is called the standard Cauchy distribution. It has the distribution of a random variable that is the ratio of two independent standard normal random variables. This has the probability density function

$$f(x;0,1)=\frac{1}{\pi(1+x^2)}$$

Its cumulative distribution function has the shape of an arctangent function arctan (x):

$$F(x;0,1)=\frac{1}{x}\arctan(x)+\frac{1}{2}$$

Johnson Su Distribution: The Johnson SU distribution is a four-parameter family of probability distributions first investigated by Johnson in 1949.It is closely related to the normal distribution.

Generation of random variables: Let U be a random variable that is uniformly distributed on the unit interval [0, 1]. Johnson SU random variables can be generated from U as follows:

$$x=\lambda\sinh\left(\frac{1}{\sigma}\Phi^{-1}(u)-\gamma\right)+x$$

where Φ is the cumulative distribution function of the normal distribution.

Normal distribution: In probability theory, the normal (or Gaussian) distribution is a continuous probability distribution, defined by the formula

$$f(x)=\frac{1}{\sigma\sqrt{s\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The parameter μ in this formula is the *mean* or *expectation* of the distribution (and also its median and mode). The parameter σ is its standard deviation; its variance is therefore σ^2. A random variable with a Gaussian distribution is said to be normally distributed and is called a normal deviate.

Logistic Distribution: In probability theory and statistics, the logistic distribution is a continuous probability distribution. Its cumulative distribution function is the logistic function, which appears in logistic regression and feed forward neural networks. It resembles the normal distribution in shape but has heavier tails (higher kurtosis).

Probability density function: The probability density function (pdf) of the logistic distribution is given by:

$$f(x;\mu,s)=\frac{e^{-\frac{x-\mu}{s}}}{s\left(1+e^{-\frac{x-\mu}{s}}\right)}=\frac{1}{4s}\sec h^2\left(\frac{x-\mu}{2s}\right)$$

Because the pdf can be expressed in terms of the square of the hyperbolic secant function "sech", it is sometimes referred to as the *sech-square(d) distribution*.

Laplace Distribution: In probability theory and statistics, the Laplace distribution is a continuous probability distribution named after Pierre-Simon Laplace. It is also sometimes called the *double exponential distribution*, because it can be thought of as two exponential (with an additional location parameter) spliced together back-to-back, but the term double exponential distribution is also sometimes used to refer to the Gumbel distribution. The difference between two independent exponential random variables is governed by a Laplace distribution, as is a Brownian motion evaluated at an exponentially distributed random time. Increments of Laplace motion or a variance gamma process evaluated over the time scale also have a Laplace distribution.

Error Function: In mathematics, the error function (also called the Gauss error function) is a special function (non-elementary) of sigmoid shape which occurs in probability, statistics and partial differential equations. It is defined as:

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2dt}$$

The complementary error function, denoted *erfc*, is defined as

$$erf(x)=1-erf(x)$$

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2dt}$$

The imaginary error function, denoted *erfi*, is defined as

$$erf(z)=-ierf(iz)$$

When the error function is evaluated for arbitrary complex arguments z, the resulting complex error function is usually discussed in scaled form as the Faddeeva function:

$$\omega(z)=e^{-z^2}erfc(-iz)$$

Hyperbolic secant distribution: In probability theory and statistics, the hyperbolic secant distribution is a continuous probability distribution whose probability density function and characteristic function are proportional to the hyperbolic secant function. The hyperbolic secant function is equivalent to the inverse hyperbolic cosine, and thus this distribution is also called the inverse-cosh distribution.

A random variable follows a hyperbolic secant distribution if its probability density function (pdf) can be related to the following standard form of density function by a location and shift transformation:

$$f(x)=\frac{1}{2}\sec h\left(\frac{\pi}{2}x\right)$$

Where "sech" denotes the hyperbolic secant function. The cumulative distribution function (cdf) of the standard distribution is

$$f(x) = \frac{1}{2} + \frac{1}{\pi}\arctan\left[\sec h\left(\frac{\pi}{2}x\right)\right]$$

$$f(x) = \frac{2}{\pi}\arctan\left[\exp\left(\frac{\pi}{2}x\right)\right]$$

where "arctan" is the inverse (circular) tangent function. The inverse cdf (or quantile function) is

$$F^{-1}(p) = -\frac{2}{\pi}ar\sinh\left[\cot\left(\pi p\right)\right]$$

$$F^{-1}(p) = \frac{2}{\pi}\ln\left[\tan\left(\frac{\pi}{2}p\right)\right]$$

where "arsinh" is the inverse hyperbolic sine function and "cot" is the (circular) cotangent function.The hyperbolic secant distribution shares many properties with the standard normal distribution: it is symmetric with unit variance and zero mean, median and mode, and its pdf is proportional to its characteristic function. However, the hyperbolic secant distribution is leptokurtic; that is, it has a more acute peak near its mean, and heavier tails, compared with the standard normal distribution.

Student's *t*-distribution: In probability and statistics, Student's t-distribution (or simply the t-distribution) is a family of continuous probability distributions that arises when estimating the mean of a normally distributed population in situations where the sample size is small and population standard deviation is unknown. It plays a role in a number of widely used statistical analyses, including the Student's t-test for assessing the statistical significance of the difference between two sample means, the construction of confidence intervals for the difference between two population means, and in linear regression analysis. The Student's t-distribution also arises in the Bayesian analysis of data from a normal family.

If we take k samples from a normal distribution with fixed unknown mean and variance, and if we compute the sample mean and sample variance for these k samples, then the distribution (for k) can be defined as the distribution of the location of the true mean, relative to the sample mean and divided by the sample standard deviation, after multiplying by the normalizing term. In this way the t-distribution can be used to estimate how likely it is that the true mean lies in any given range.

The t-distribution is symmetric and bell-shaped, like the normal distribution, but has heavier tails, meaning that it is more prone to producing values that fall far from its mean. This makes it useful for understanding the statistical behavior of certain types of ratios of random quantities, in which variation in the denominator is amplified and may produce outlying values when the denominator of the ratio falls close to zero. The Student's t-distribution is a special case of the generalised hyperbolic distribution.

Results and discussion

Table-1 visualises the result of descriptive that statistics shows the Minimum, Mean, Maximum, Standard Deviation, Covariance, The average return of FTSE100 is higher when comparative to the average return of other G7 countries. The result of Standard Deviation of shows Italy has higher volatility when compared to other selected countries. Table-2 visualizes the parameter of different unbounded distribution fitted for Nikkei. In table-3 the goodness of fit is shown for different distribution fitted for returns of Nikkei. The Johnson Su distribution has the best fit when compared to other unbounded distributions. Table-4 shows the parameter of different unbounded distribution fitted for Nikkei. In table-5 the goodness of fit is shown for different distribution fitted for returns of NASDAQ. The Johnson Su distribution has the best fit in compared to other unbounded distributions. Table-6 shows visualize the parameter of different unbounded distribution fitted for Nikkei. In of table-7 the goodness of fit is shown for different distribution fitted for returns of CAC 40. The Gumbel Min distribution has the best fit in compared to other unbounded distributions. Table-8 shows visualize the parameter of different unbounded distribution fitted for Nikkei. In table-9 the goodness of fit is shown for different distribution fitted for returns of DBE. The Johnson Su distribution has the best fit in compared to other unbounded distributions. Table-10 shows visualize the parameter of different unbounded distribution fitted for Nikkei. In of table-11 the goodness of fit is shown for different distribution fitted for returns of FTSE.MIB. The Cauchy distribution has the best fit in compared to other unbounded distributions. Table-12 shows visualize the parameter of different unbounded distribution fitted for Nikkei. In of table-13 the goodness of fit is shown for different distribution fitted for returns of FTSE 100. The Gumbel min distribution has the best fit in compared to other unbounded distributions.

Table-1
Descriptive Statistics

Country Index	Minimum	Mean	Maximum	Standard Deviation	C.V
NIKKEI	-23.8269	0.3764	12.8499	5.7359	15.2357
FTSE.MIB	-16.3063	1.0956	155.130	15.5759	14.2159
NASDAQ	-17.7319	0.7133	12.3454	5.2211	7.3194
FTSE100	-13.0238	0.4710	8.4533	3.9129	8.3069
DBE	-19.1921	0.9093	16.7621	5.4483	5.9914
CAC40	-13.5173	0.2808	12.5567	4.8376	17.2275

Table-2
Maximum Likelihood Estimates of Unbounded Distribution Parameters for NIKKEI

Unbounded distribution	Parameters(Scale, Shape, Allocation)								
	σ	μ	K	h	λ	δ	ξ	γ	ν
Cauchy	3.0377	0.76695	-	-	-	-	-	-	-
Error	5.7359	0.37648	1.1516	-	-	-	-	-	-
Error Function	-	-	-	0.12328	-	-	-	-	-
Gumbel Max	4.4723	2.205	-	-	-	-	-	-	-
Gumbel Min	4.4723	2.9579	-	-	-	-	-	-	-
Hypersecant	5.7359	0.37648	-	-	-	-	-	-	-
Johnson SU	-	-	-	-	9.9061	2.1882	6.0448	1.0832	-
Laplace	-	0.37648	-	-	-	-	-	-	-
Logistic	3.1624	0.37648	-	-	-	-	-	-	-
Normal	5.7359	0.37648	-	-	-	-	-	-	-
Student's t	-	-	-	-	-	-	-	-	2

Table-3
Goodness of Fit – Summary

Unbounded distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
	Statistic	Rank	Statistic	Rank	Statistic	Rank
Cauchy	0.08699	6	1.186	9	2.2751	1
Error	0.06639	3	0.47786	4	3.1663	4
Error Function	0.11376	9	1.1236	8	8.857	9
Gumbel Max	0.14915	10	4.6355	10	10.199	10
Gumbel Min	0.09195	8	0.98342	7	4.329	6
Hypersecant	0.06608	2	0.43631	3	2.5044	3
Johnson SU	0.06153	1	0.24798	1	4.4021	7
Laplace	0.08181	5	0.71043	6	4.1699	5
Logistic	0.07602	4	0.39937	2	2.2806	2
Normal	0.08811	7	0.60059	5	5.7298	8
Student's t	0.30987	11	39.103	11	111.66	11

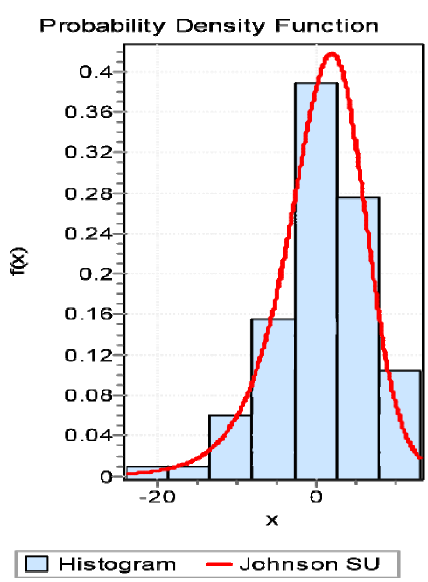

Figure-1

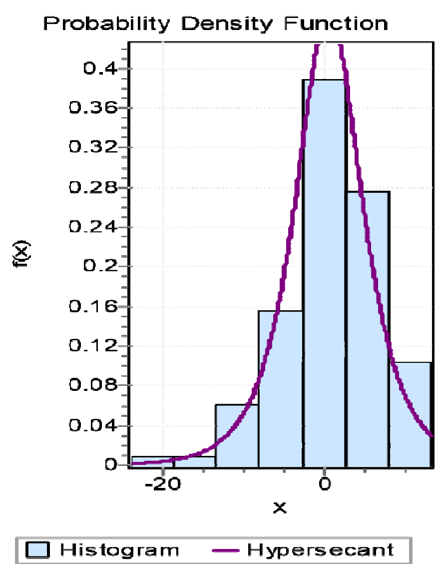

Figure-2

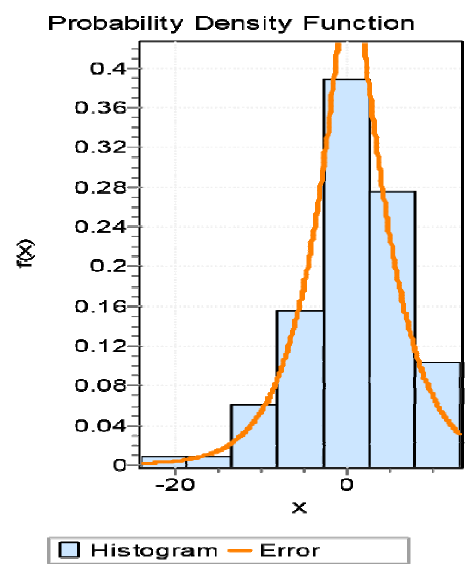

Figure-3

Figure showing Probability density function for NIKKEI

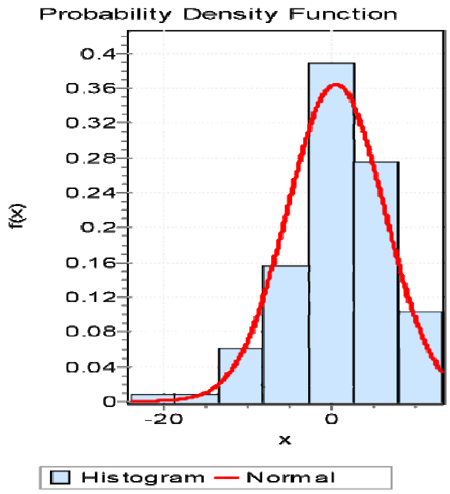

Figure-4

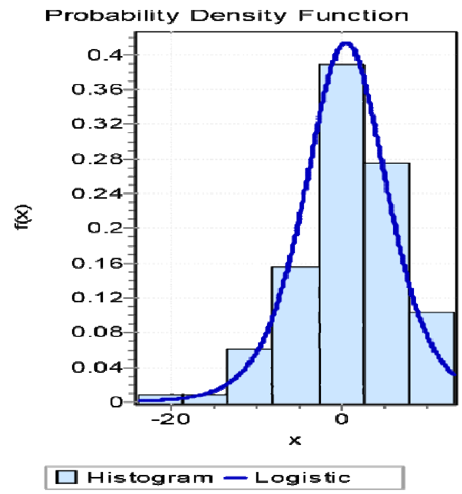

Figure-5

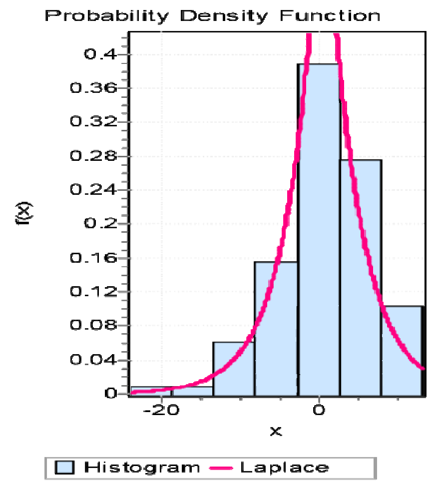

Figure-6

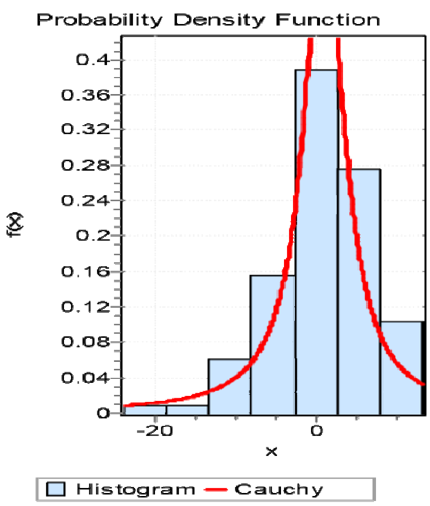

Figure-7

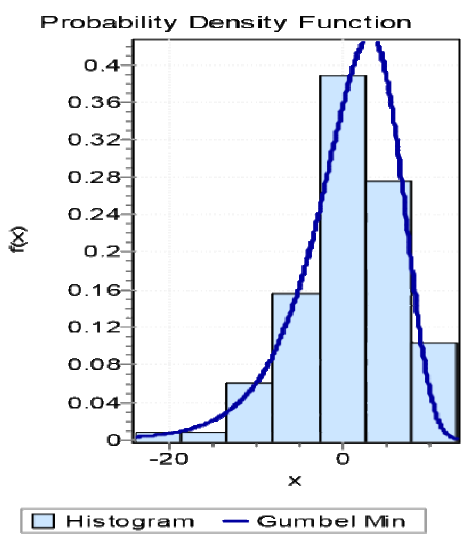

Figure-8

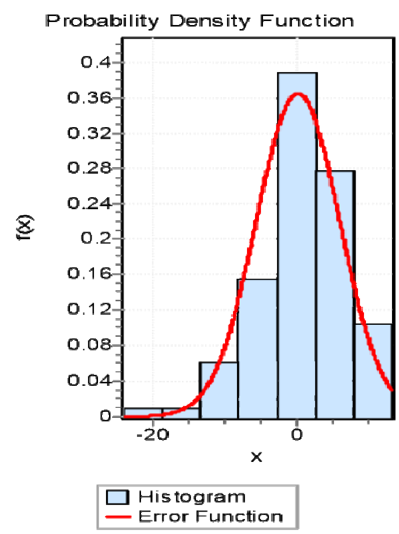

Figure-9

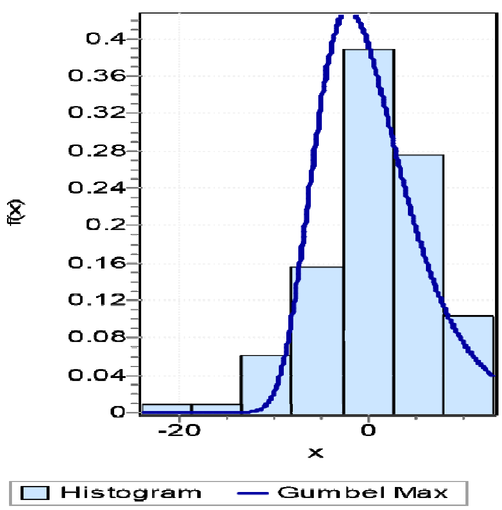

Figure-10

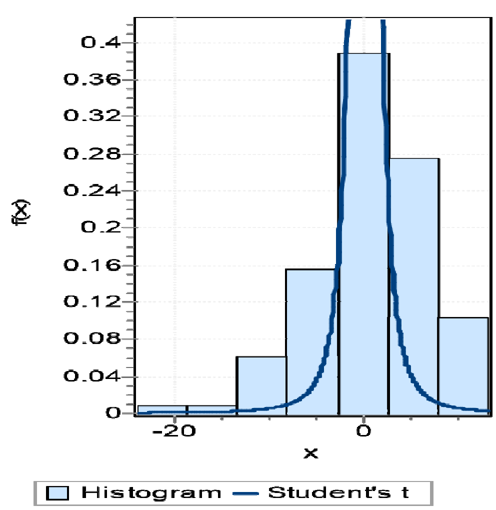

Figure-11

Figure showing Probability density function for NIKKEI

Table-4
Maximum Likelihood Estimates of Unbounded Distribution Parameters for NASDAQ

Unbounded distribution	Parameters(Scale, Shape, Allocation)								
	σ	μ	K	h	λ	δ	ξ	γ	ν
Cauchy	3.0862	1.4813	-	-	-	-	-	-	-
Error	5.2211	0.71332	1.5355	-	-	-	-	-	-
Error Function	-	-	-	0.13543	-	-	-	-	-
Gumbel Max	4.0709	1.6365	-	-	-	-	-	-	-
Gumbel Min	4.0709	3.0631	-	-	-	-	-	-	-
Hypersecant	5.2211	0.71332	-	-	-	-	-	-	-
Johnson SU	-	-	-	-	14.865	3.7332	12.33	2.5912	-
Laplace	-	0.71332	-	-	-	-	-	0.27086	-
Logistic	2.8786	0.71332	-	-	-	-	-	-	-
Normal	5.2211	0.71332	-	-	-	-	-	-	-
Student's t	-	-	-	-	-	-	-	-	2

Table-5
Goodness of Fit – Summary

Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
	Statistic	Rank	Statistic	Rank	Statistic	Rank
Cauchy	0.10386	7	2.1934	8	9.3559	4
Error	0.08828	4	0.7959	4	9.5098	5
Error Function	0.11907	8	2.3751	9	10.789	7
Gumbel Max	0.13241	10	4.8265	10	19.346	10
Gumbel Min	0.07362	3	0.74403	2	3.9459	1
Hypersecant	0.10343	6	1.0569	6	12.283	8
Johnson SU	0.05035	1	0.32622	1	6.9946	2
Laplace	0.1306	9	1.8301	7	18.511	9
Logistic	0.08859	5	0.80796	5	8.5055	3
Normal	0.06924	2	0.75442	3	10.761	6
Student's t	0.36869	11	47.473	11	72.473	11

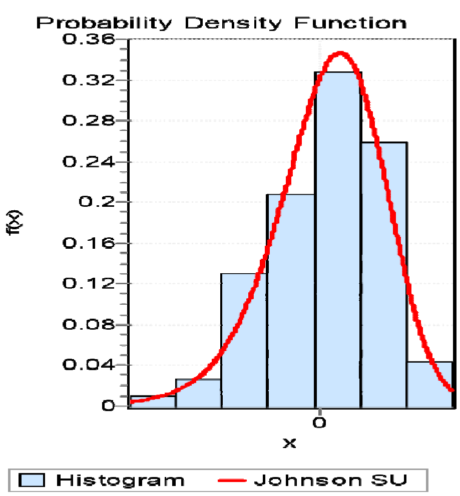

Figure-12

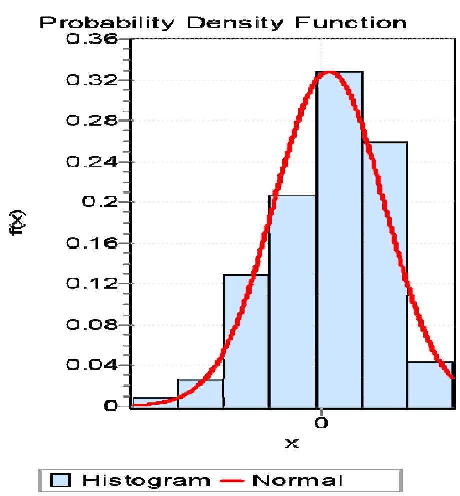

Figure-13

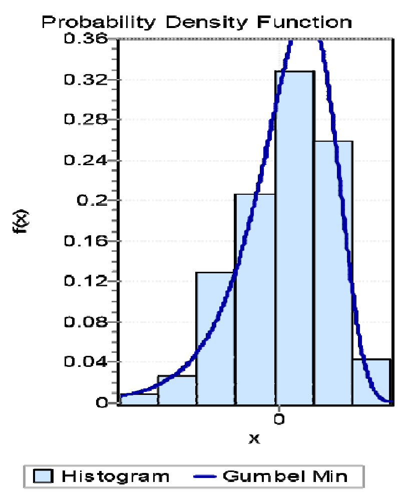

Figure-14

Figure showing Probability density function for NASDAQ

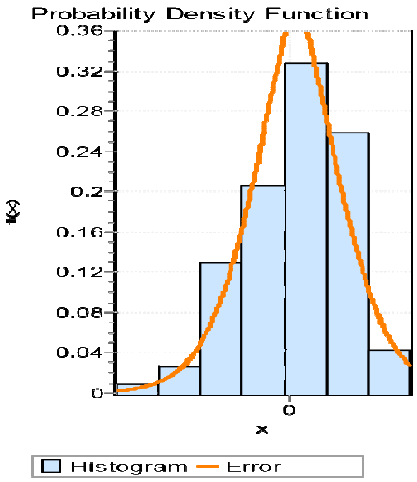

Figure-15

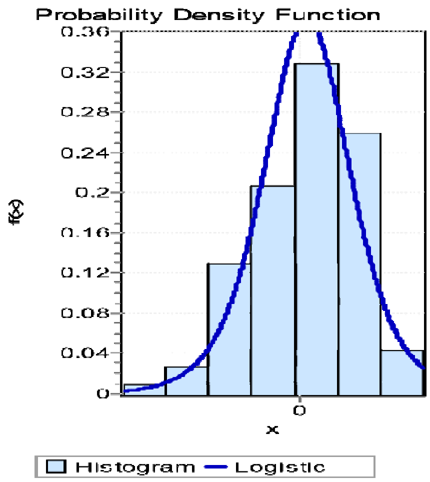

Figure-16

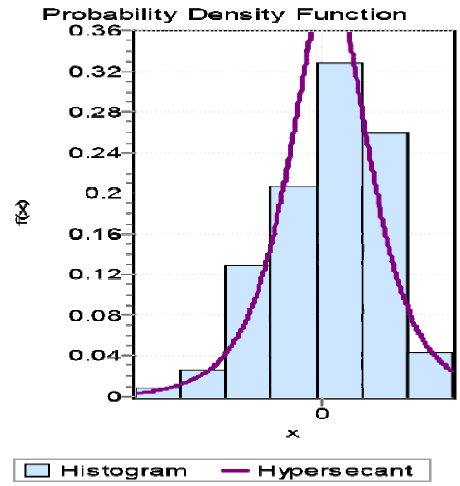

Figure-17

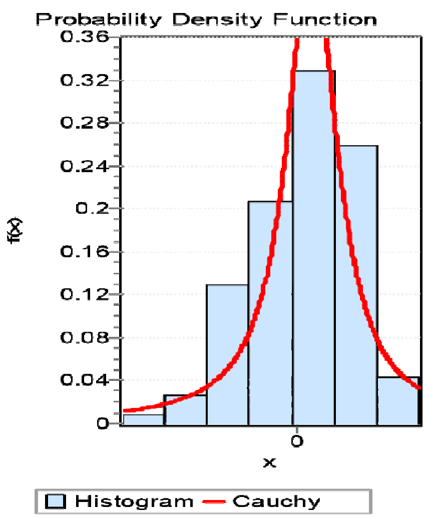

Figure-18

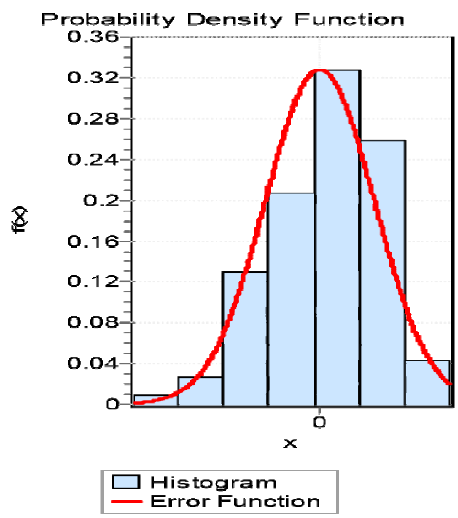

Figure-19

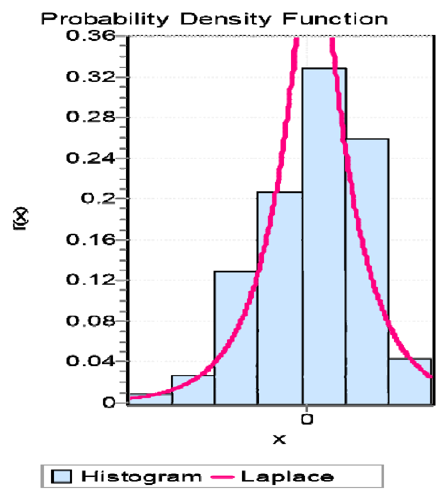

Figure-20

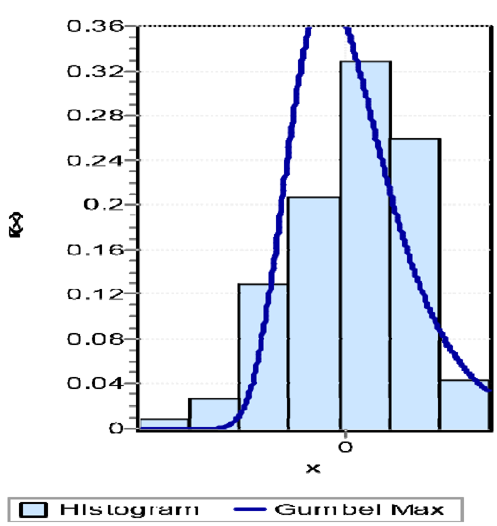

Figure-21

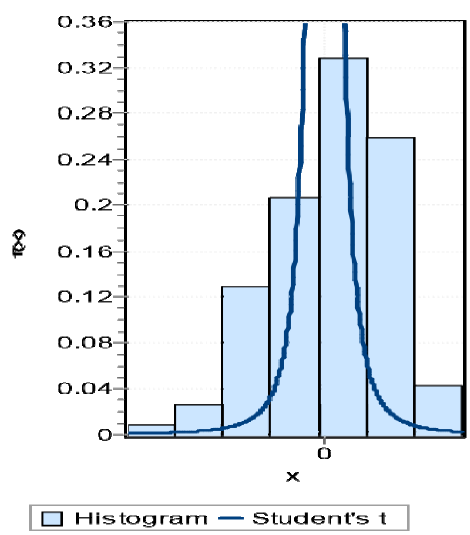

Figure-22

Figure showing Probability density function for NASDAQ

Table-6
Maximum Likelihood Estimates of Unbounded Distribution Parameters for CAC40

Unbounded distribution	Parameters(Scale, Shape, Allocation)						
	σ	μ	K	h	λ	δ	ξ
Cauchy	2.6209	1.4276	-	-	-	-	-
Error	4.8376	0.28081	1.6357	-	-	-	-
Error Function	-	-	-	0.14617	-	-	-
Gumbel Max	3.7719	1.8964	-	-	-	-	-
Gumbel Min	3.7719	2.458	-	-	-	-	-
Hypersecant	4.8376	0.28081	-	-	-	-	-
Johnson SU	-	-	-	-	-	-	-
Laplace	-	0.28081	-	-	0.29234	-	-
Logistic	2.6671	0.28081	-	-	-	-	-
Normal	4.8376	0.28081	-	-	-	-	-
Student's t	-	-	-	-	-	-	2

Table-7
Goodness of Fit – Summary

Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
	Statistic	Rank	Statistic	Rank	Statistic	Rank
Cauchy	0.10744	5	2.6428	8	11.786	4
Error	0.10629	3	1.2378	2	10.334	3
Error Function	0.1235	7	1.8018	6	12.005	5
Gumbel Max	0.17075	9	6.6645	9	23.917	9
Gumbel Min	0.04252	1	0.31413	1	2.3208	1
Hypersecant	0.11278	6	1.4884	5	12.396	6
Laplace	0.13859	8	2.3391	7	15.11	8
Logistic	0.10721	4	1.2507	3	9.7261	2
Normal	0.10062	2	1.2632	4	14.156	7
Student's t	0.3533	10	42.053	10	158.88	10
Johnson SU	No fit					

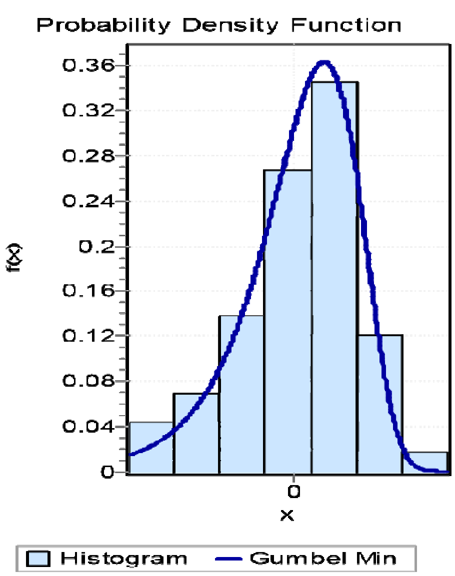

Figure-23

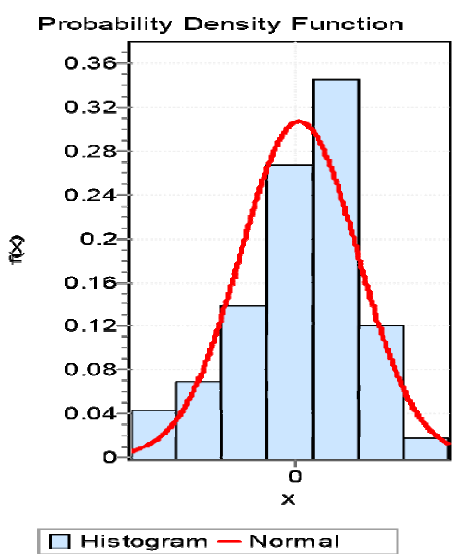

Figure-24

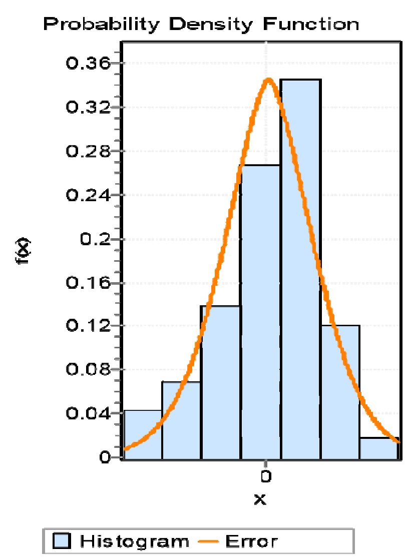

Figure-25

Figure showing Probability density function for CAC40

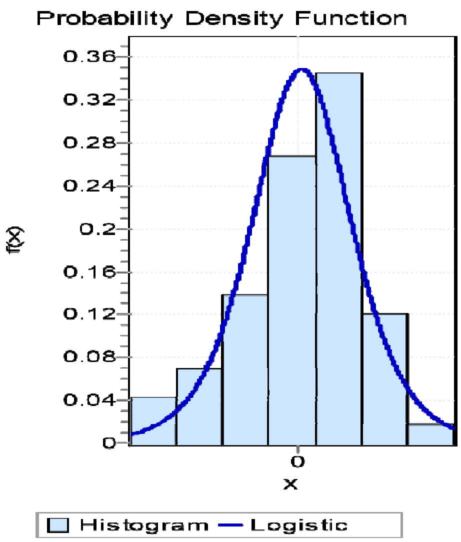

Figure-26

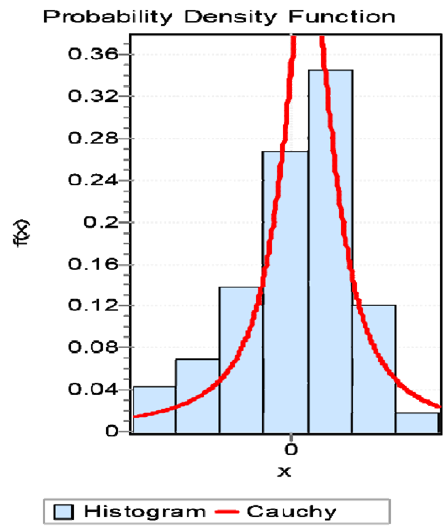

Figure-27

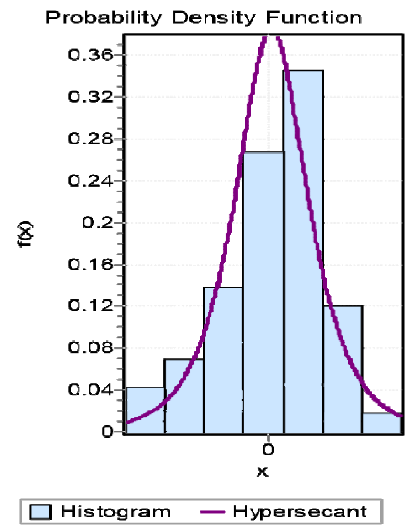

Figure-28

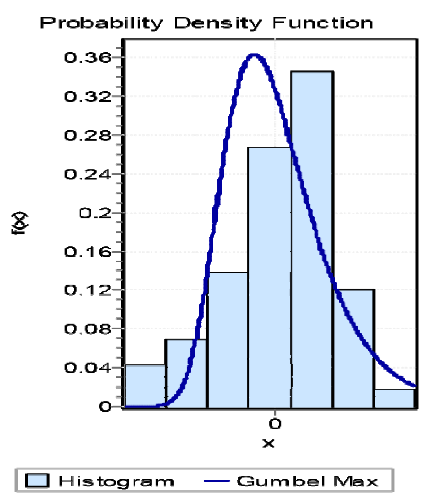

Figure-29

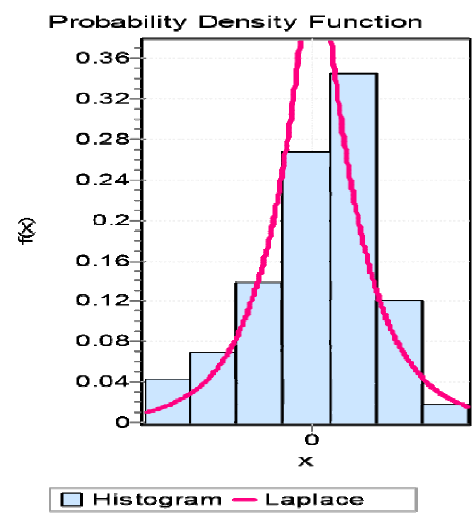

Figure-30

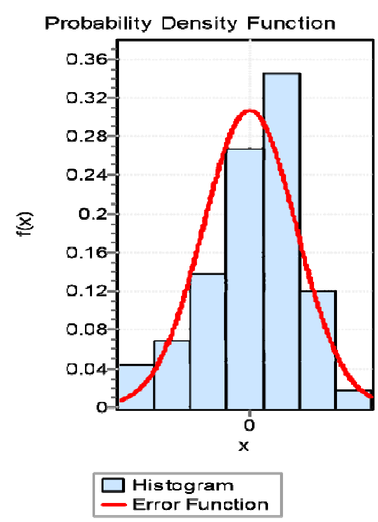

Figure-31

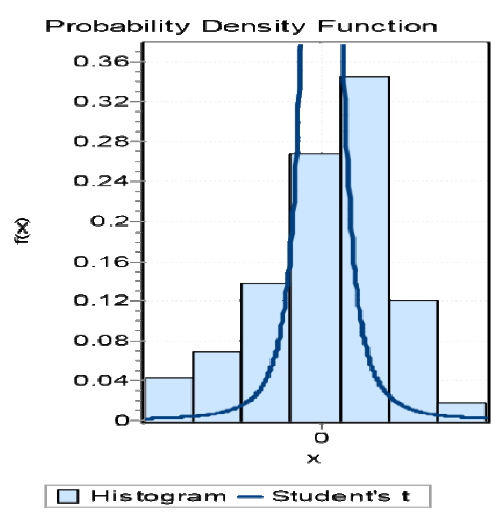

Figure-32

Figure showing Probability density function for CAC40

Table-8
Maximum Likelihood Estimates of Unbounded Distribution Parameters for DBE

Unbounded distribution	Parameters(Scale, Shape, Allocation)								
	σ	μ	K	h	λ	δ	ξ	γ	ν
Cauchy	2.575	1.9164	-	-	-	-	-	-	-
Error	5.4483	0.90935	1.1355	-	-	-	-	-	-
Error Function	-	-	-	0.12978	-	-	-	-	-
Gumbel Max	4.248	1.5427	-	-	-	-	-	-	-
Gumbel Min	4.248	3.3614	-	-	-	-	-	-	-
Hypersecant	5.4483	0.90935	-	-	-	-	-	-	-
Johnson SU	-	-	-	-	9.0834	2.0531	5.0713	0.81402	-
Laplace	-	0.90935	-	-	0.25957	-	-	-	-
Logistic	3.0038	0.90935	-	-	-	-	-	-	-
Normal	5.4483	0.90935	-	-	-	-	-	-	-
Student's t	-	-	-	-	-	-	-	-	2

Table-9
Goodness of Fit - Summary

Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
	Statistic	Rank	Statistic	Rank	Statistic	Rank
Cauchy	0.09862	3	1.9678	8	13.255	5
Error	0.12871	7	1.1639	4	13.067	4
Error Function	0.16014	9	4.0859	9	14.114	6
Gumbel Max	0.16463	10	6.3349	10	17.242	9
Gumbel Min	0.09951	4	1.2954	5	4.8304	1
Hypersecant	0.11531	6	1.0263	2	15.075	7
Johnson SU	0.06951	1	0.5883	1	8.2325	2
Laplace	0.14006	8	1.3676	6	8.5474	3
Logistic	0.10665	5	1.1207	3	16.345	8
Normal	0.09623	2	1.632	7	20.331	10
Student's t	0.42517	11	51.854	11	181.14	11

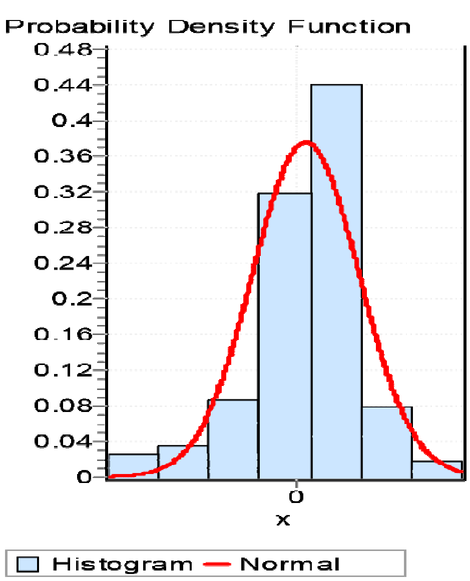

Figure-33

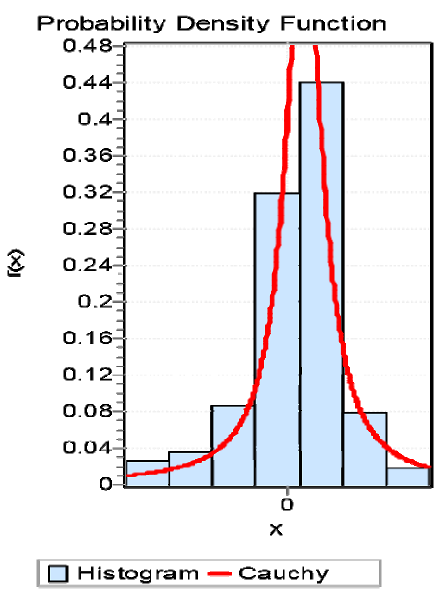

Figure-34

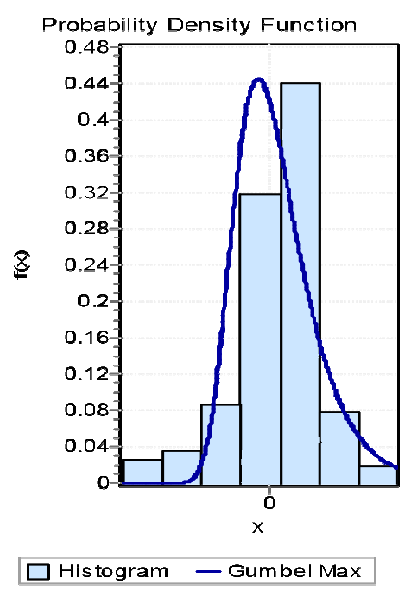

Figure-35

Figure showing Probability density function for DBE

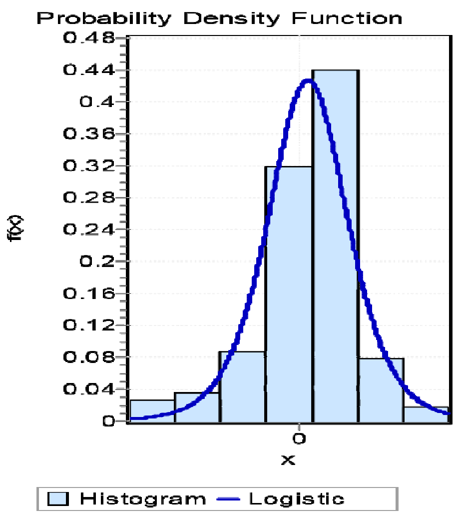

Figure-36

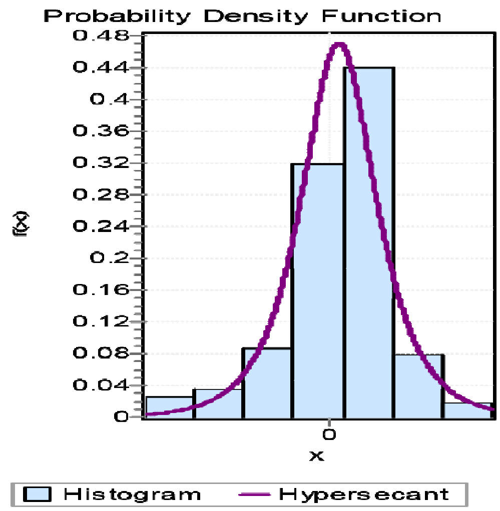

Figure-37

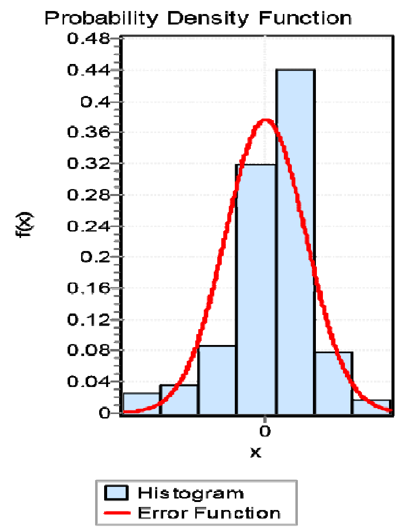

Figure-38

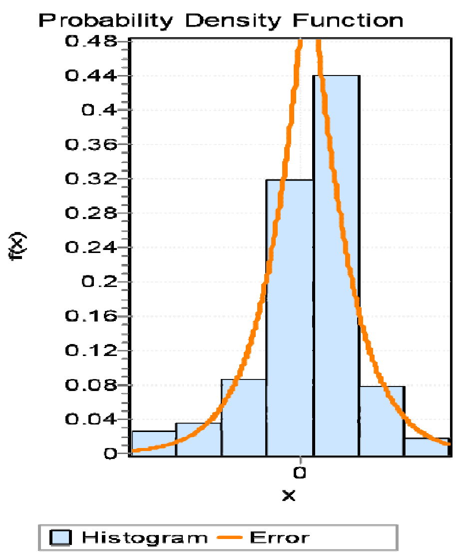

Figure-39

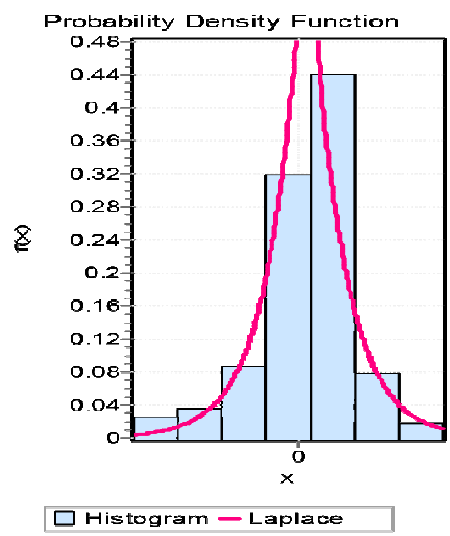

Figure-40

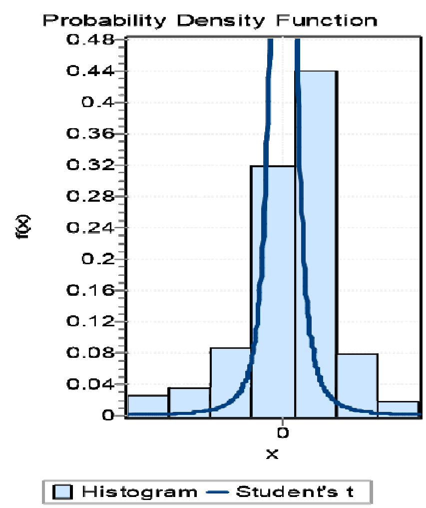

Figure-41

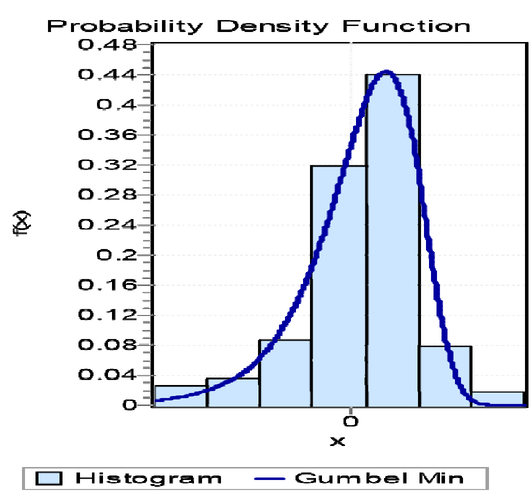

Figure-42

Figure showing Probability density function for DBE

Table-10
Maximum Likelihood Estimates Of Unbounded Distribution Parameters for FTSE.MIB

Unbounded distribution	**Parameters(Scale, Shape, Allocation)**						
	σ	**μ**	**K**	**h**	**λ**	**δ**	**ξ**
Cauchy	3.3521	0.69134	-	-	-	-	-
Error	15.576	1.0957	1.0	-	-	-	-
Error Functio n	-	-	-	0.0454	-	-	-
Gumbel Max	12.144	5.9143	-	-	-	-	-
Gumbel Min	12.144	8.1057	-	-	-	-	-
Hypersecant	15.576	1.0957	-	-	-	-	-
Johnson SU	-	-	-	-	-	-	-
Laplace	-	1.0957	-	-	0.09079	-	-
Logistic	8.5875	1.0957	-	-	-	-	-
Normal	15.576	1.0957	-	-	-	-	-
Student's t	-	-	-	-	-	-	2

Table-11
Goodness of Fit – Summary

Distribution	**Kolmogorov Smirnov**		**Anderson Darling**		**Chi-Squared**	
	Statistic	**Rank**	**Statistic**	**Rank**	**Statistic**	**Rank**
Cauchy	0.09227	1	1.9327	1	8.6511	1
Error	0.22488	3	9.7473	3	59.192	3
Error Function	0.24926	5	17.539	7	119.8	8
Gumbel Max	0.24089	4	14.112	6	96.652	6
Gumbel Min	0.33484	10	22.804	9	N/ A	
Hypersecant	0.25167	6	12.412	4	84.805	4
Laplace	0.22488	2	9.7473	2	59.192	2
Logistic	0.26191	7	13.856	5	94.421	5
Normal	0.27445	8	17.883	8	119.26	7
Student's t	0.33385	9	47.772	10	178.73	9
Johnson SU	No fit					

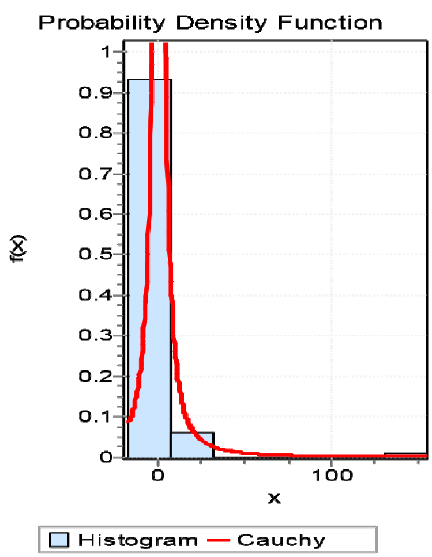

Figure-43

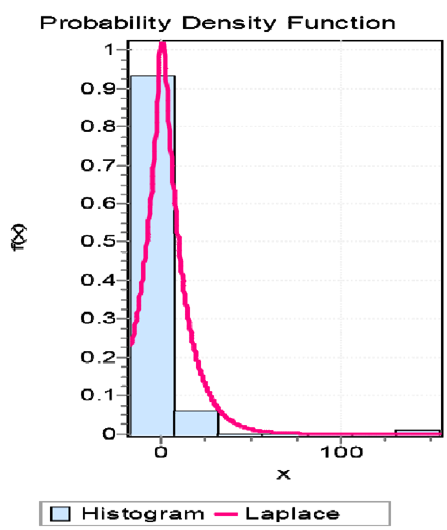

Figure-44

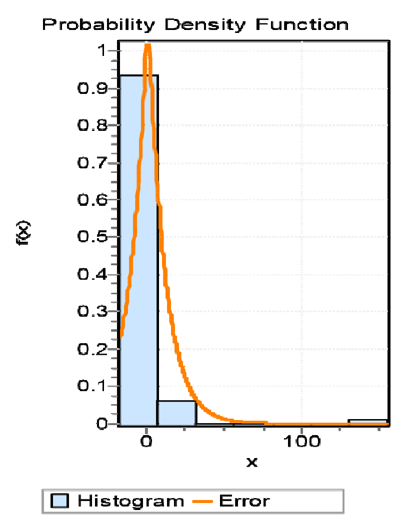

Figure-45

Figure showing Probability density function for FTSE.MIB

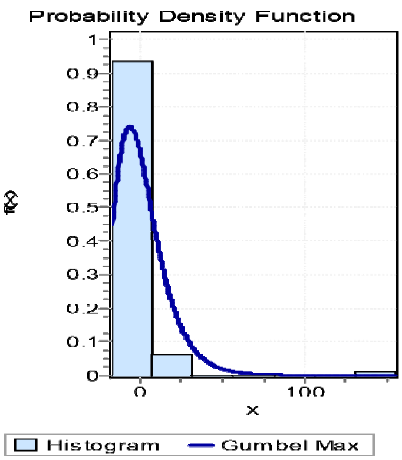

Figure-46

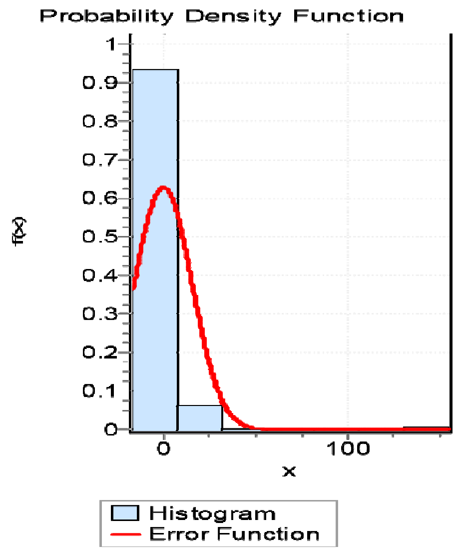

Figure-47

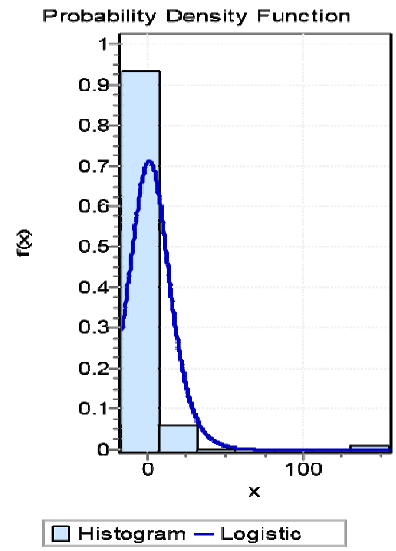

Figure-48

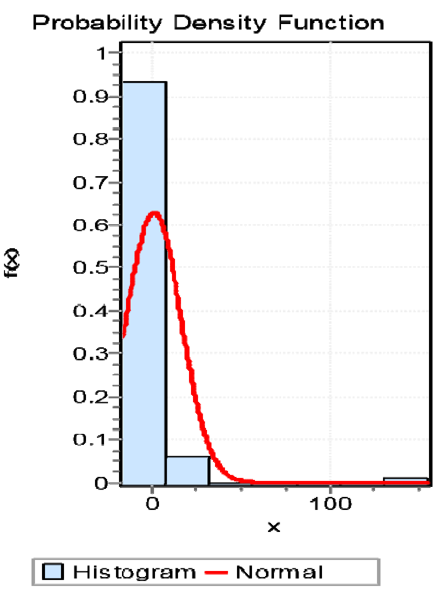

Figure-49

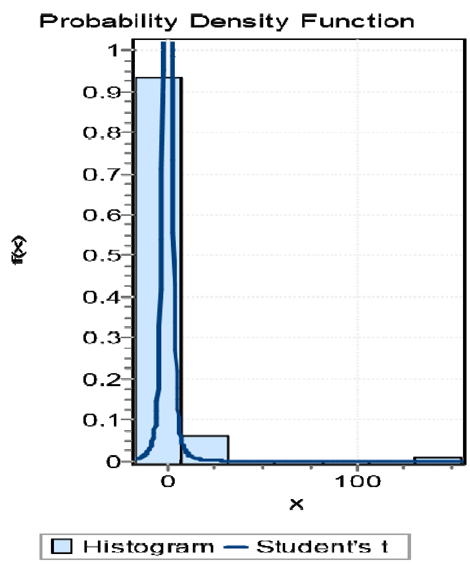

Figure-50

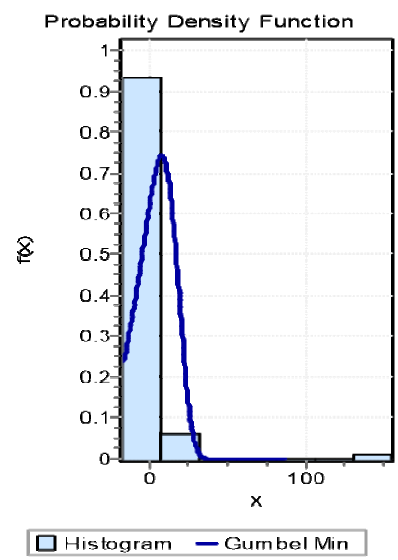

Figure-51

Figure showing Probability density function for FTSE.MIB

Table-12
Maximum Likelihood Estimates Of Unbounded Distribution Parameters for FTSE100

Unbounded distribution	Parameters(Scale, Shape, Allocation)								
	σ	μ	K	h	λ	δ	ξ	γ	ν
Cauchy	2.0769	1.1541	-	-	-	-	-	-	-
Error	3.9129	0.47104	-	-	-	-	-	-	-
Error Function	-	-	-	0.18071	-	-	-	-	-
Gumbel Max	3.0509	1.29	-	-	-	-	-	-	-
Gumbel Min	3.0509	2.2321	-	-	-	-	-	-	-
Hypersecant	3.9129	0.47104	-	-	-	-	-	-	-
Johnson SU	-	-	-	-	8.2544	3.579	11.318	3.7802	-
Laplace	-	0.47104	-	-	0.36142	-	-	-	-
Logistic	2.1573	0.47104	-	-	-	-	-	-	-
Normal	3.9129	0.47104	-	-	-	-	-	-	-
Student's t	-	-	-	-	-	-	-	-	2

Table-13
Goodness of Fit – Summary

Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
	Statistic	Rank	Statistic	Rank	Statistic	Rank
Cauchy	0.08824	6	1.8974	8	14.212	7
Error	0.08694	5	0.93782	4	13.818	6
Error Function	0.12268	9	2.6473	9	15.35	8
Gumbel Max	0.14534	10	6.1246	10	25.241	10
Gumbel Min	0.0741	1	0.86617	2	1.9334	1
Hypersecant	0.0942	7	0.95409	5	11.286	5
Johnson SU	0.07632	3	0.49845	1	2.2823	2
Laplace	0.12141	8	1.4745	7	17.302	9
Logistic	0.07938	4	0.90449	3	8.6752	4
Normal	0.07481	2	1.1601	6	8.6119	3
Student's t	0.31599	11	28.959	11	106.53	11

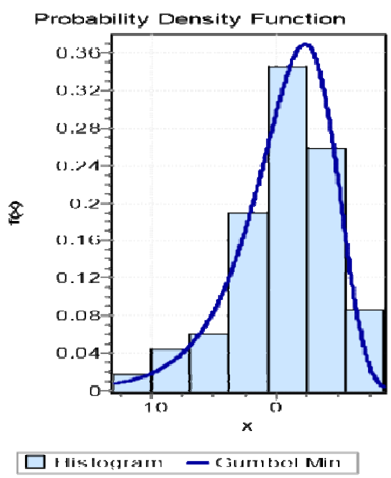

Figure-52

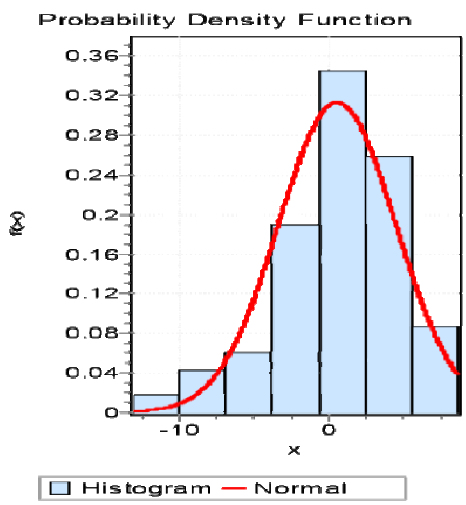

Figure-53

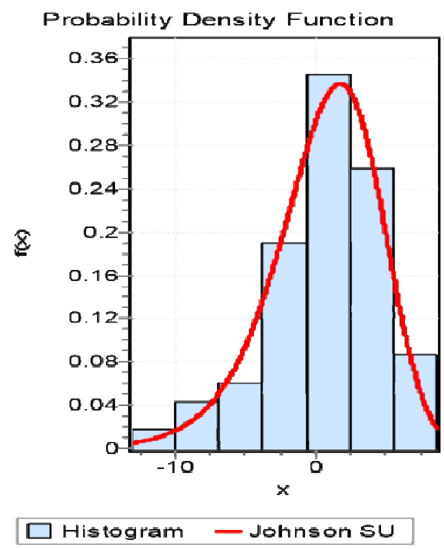

Figure-54

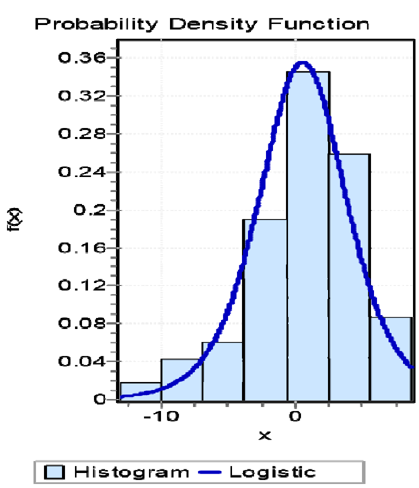

Figure-55

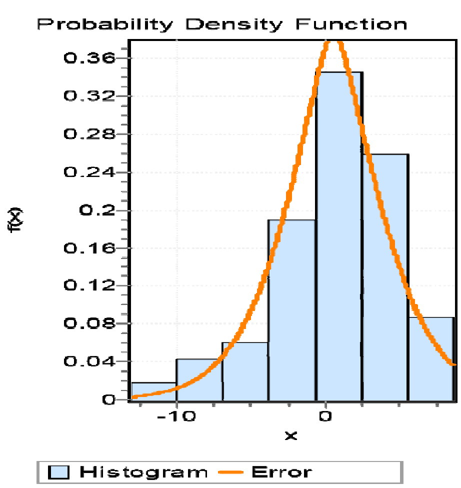

Figure-56

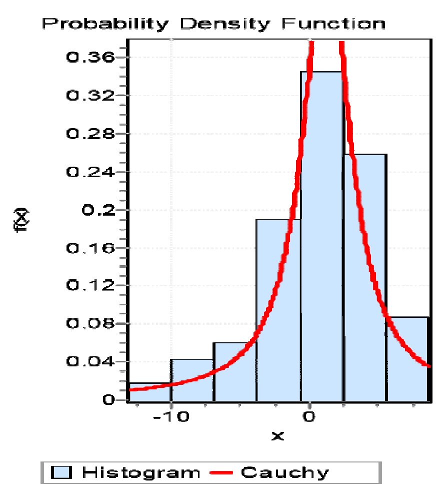

Figure-57

Figure showing Probability density function for FTSE 100

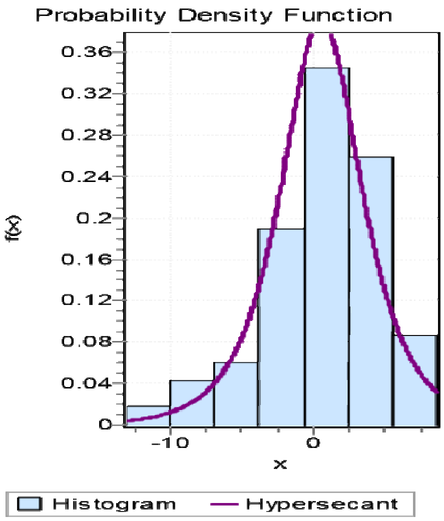

Figure-58

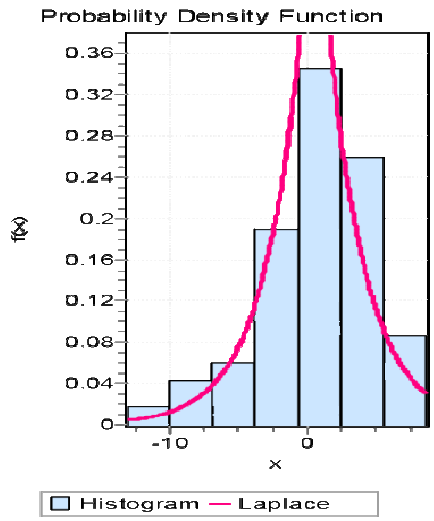

Figure-59

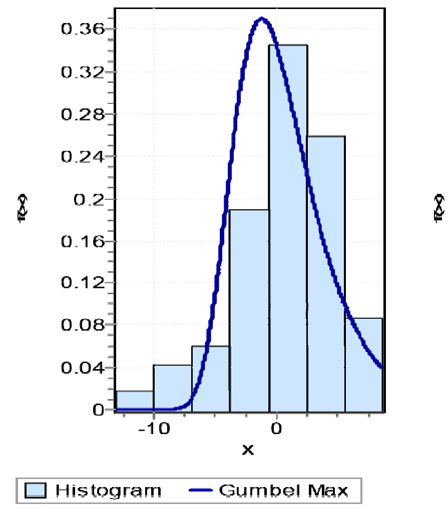

Figure-60

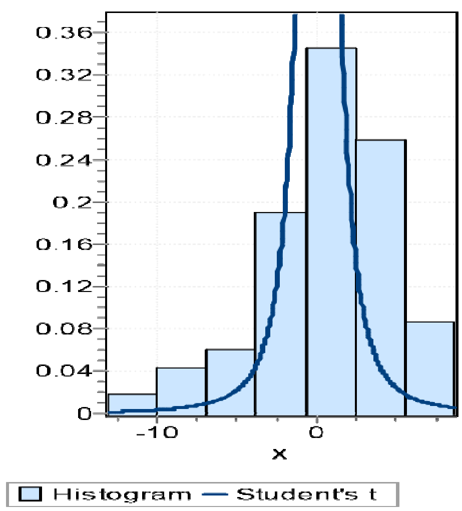

Figure-61

Figure showing Probability density function for FTSE 100

Conclusion

From the above made detailed analysis for the selected index of the countries namely U.S, U.K, GERMAN, ITALY, FRANCE, JAPAN John Su distributions is well suited for the following index NIKKEI, NASDAQ, DBE. While CAC40 and FTSE 100 follows Gumbel min distribution and finally, FTSE MIB follows Cauchy distributions. When the analysis carried out using the above mentioned unbounded distribution for particular index then there is a good probability for achieving an accurate result. As the returns of the indices were considered for analysis so it will be appropriate to use unbounded distributions alone for the study.

References

1. Bachelier L., Theorie de la speculation, *Ann. Sci. Ec. Norm. Super,* **111117,** 2186 **(1890)**
2. Black F. and Scholes M., The pricing of options and corporate liabilities, *The journal of political economy*, 637-654 **(1973)**
3. Bouchaud J.P. and Potters M., *Theory of financial risk and derivative pricing: from statistical physics to risk management*, Cambridge University Press **(2003)**
4. Jansen D.W. and De Vries C.G., on the frequency of large stock returns: Putting booms and busts into perspective, *The review of economics and statistics*, 18-24 **(1991)**
5. Krawiecki A., Hołyst J.A. And Helbing D., Volatility clustering and scaling for financial time series due to attractor bubbling, *Physical review letters*, **89(15),** 158701 **(2002)**
6. Kullmann L., Töyli J., Kertesz J., Kanto A. and andKaski K., Characteristic times in stock market indices, *Physica A: Statistical Mechanics and its Applications*, **269(1),** 98-110 **(1999)**
7. Liu Y., Cizeau P., Meyer M., Peng C. K. and Eugene Stanley H., Correlations in economic time series, *Physica A: Statistical Mechanics and its Applications*, **245(3),** 437-440 **(1997)**
8. Mantegna R.N. and Stanley H.E., Stochastic process with ultraslow convergence to a Gaussian: the runcated Lévy flight, *Physical Review Letters*, **73(22),** 2946 **(1994)**
9. Mantegna R.N. and Stanley H.E., An Introduction to Econophysics, Correlations and Complexity in Finance, Cambridge University Press, Cambridge **(1999)**
10. Mandelbrot B.B., The variation of certain speculative prices, (371-418) Springer New York **(1997)**