Research Journal of Management Sciences ISSN 2319–1171
Vol. **1(2),** 15-23, September **(2012)** *Res. J. Management Sci.*

A New Generalisation of Sam-Solai's Multivariate Additive Beta Distribution of Kind-2 of Type-A

David G.S., Sam Jayakumar, Solairaju A. and Dawood Ali S.,
Assistant Professor, Jamal Institute of Management, Tiruchirappalli, South India, INDIA
Associate Professor, Dept. Of Mathematics, Jamal Mohamed college, Tiruchirappalli, South India, INDIA
Assistant Professor, Jamal Institute of Management, Tiruchirappalli – 620 020 South India, INDIA

Available online at: www.isca.in
Received 28th August 2012, revised 30th August 2012, accepted 3rd September 2012

Abstract

This paper proposed a new generalization of bounded continuous multivariate symmetric probability distributions. More specifically the authors visualizes a new generalization of Sam-Solai's multivariate additive Beta distribution of Kind-2 of Type-A from the uni-variate two parameter Beta distribution of Kind-1. Further,we find its marginal, multivariate conditional distributions, multivariate generating functions, multivariate survival, hazard functions and also discussed it's special cases. The special cases includes the transformation of Sam-Solai's multivariate additive Beta distribution of Kind -2 of Type-A into multivariate additive Beta distribution of Kind-1 of Type-A, Multivariate F-distribution of Kind-1, Multivariate standard Logistic-Beta distribution of Kind-1. Moreover, it is found that the bivariate correlation between two Beta random variables purely depends on the shape parameter and we simulated and established selected standard bivariate Beta correlation bounds from 10,000 different combinations of values for shape parameter.

Keywords: Sam-Solai's multivariate additive beta distribution, transformation, multivariate additive beta distribution of kind-1 of type-a, multivariate f-distribution of kind-1, multivariate standard logistic-beta distribution of kind-1, correlation bounds

Introduction

The origin of Beta distribution and its multivariate generalization was extensively studied because of the wide applications of the distribution in various fields for the past five decades.Dirichlet generalization of the multivariate beta distribution is a constrained form and it is an extension of multinomial distribution to the Continuous case. Many authors attempted to give alternate form of multivariate generalization of Beta distribution. The non-central multivariate beta distribution to study the distribution of the multiple correlation matrix[1] and the generalized form of the multivariate beta distribution[2]. The Multivariate Generalized beta distribution with special reference to the utility assessment theories[3]. The generalized matric variate beta distribution[4] and The multivariate F and beta distribution for the purpose of finding the exact moments of the distributions[5]. The beta and gamma matrices, singular wishart type beta distributions matrix variate beta distributions and some applications of the beta distributions[6,7,8,9]. The Multivariate t and beta distributions are closely associated with the multivariate F distribution[10] and the matrix variate version of the Kummer Beta distribution[11]. The beta distribution to the multivariate case in an in-depth manner[12,13,14]. From the in-depth reviews of beta distribution, this paper proposed an alternate form of multivariate beta distribution and its structure, form were discussed in the next section.

Section 1: Sam-Solai's Multivariate Additive Beta distribution, Definition 1.1: Let $X_1, X_2, X_3 \ldots, X_p$ are the random variables followed Continuous univariate beta distribution of second kind with shape parameters (a_i, b_i) for all i (*i=1 to p*), then the Multivariate Sam-Solai's additive beta distribution of kind-2 of Type-A and its density function is defined as

$$f(x_1,x_2,x_3\ldots,x_p)=\{(\sum_{i=1}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1})-(p-1)\}\prod_{i=1}^{p}\frac{1}{B(1,b_i)(1+x_i)^{b_i+1}} \quad (1)$$

Where $0 \le x_i \le \infty,\ a_i, b_i > 0$

*Mathematics Subject Classification. Primary 62H10; Secondary 62E15

Theorem 1.2: The cumulative distribution function of the Sam-Solai's Multivariate additive Beta distribution is defined by

$$F(x_1,x_2,x_3\ldots,x_p)=\int_0^{x_1}\int_0^{x_2}\int_0^{x_3}\cdots\int_0^{x_p}\{(\sum_{i=1}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{u_i}{1+u_i})^{a_i-1})-(p-1)\}\prod_{i=1}^{p}\frac{1}{B(1,b_i)(1+u_i)^{b_i+1}}du_i \quad (2)$$

Where, $0\le u_i<x_i$, $a_i,b_i>0$

$$F(x_1,x_2,x_3\ldots,x_p)=\{(\sum_{i=1}^{p}\frac{B(1,b_i)b_i(1+x_i)^{b_i}}{((1+x_i)^{b_i}-1)}\int_0^{x_i}\frac{u_i^{a_i-1}}{B(a_i,b_i)(1+u_i)^{a_i+b_i}}du_i)-(p-1)\}\prod_{i=1}^{p}\frac{((1+x_i)^{b_i}-1)}{B(1,b_i)b_i(1+x_i)^{b_i}}$$

$$F(x_1,x_2,x_3\ldots,x_p)=\{(\sum_{i=1}^{p}\frac{(1+x_i)^{b_i}\phi_i(x_i;a_i,b_i)}{((1+x_i)^{b_i}-1)})-(p-1)\}\prod_{i=1}^{p}\frac{((1+x_i)^{b_i}-1)}{(1+x_i)^{b_i}}$$

where $\phi_i(x_i;a_i,b_i)=\int_0^{x_i}\frac{u_i^{a_i-1}}{B(a_i,b_i)(1+u_i)^{a_i+b_i}}du_i$ is the lower incomplete Beta integral of i^{th} random variable.

Theorem 1.3: The Probability density function of Sam-Solai's Multivariate additive Conditional Beta distribution of X_1 on $X_2,X_3,\ldots X_P$ is

$$f(x_1/x_2,x_3\ldots,x_p)=\frac{\{(\sum_{i=1}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1})-(p-1)\}\frac{1}{B(1,b_1)(1+x_1)^{b_1+1}}}{\{(\sum_{i=2}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1})-(p-2)\}} \quad (3)$$

Where $0\le x_1\le\infty$, $a_i,b_i>0$

Proof: It is obtained from

$$f(x_1/x_2,x_3\ldots,x_p)=\frac{f(x_1,x_2,x_3\ldots,x_p)}{f(x_2,x_3\ldots,x_p)}$$

Theorem 1.4: Mean and Variance of Sam - Solai's Multivariate additive Conditional Beta distribution are

$$E(x_1/x_2,x_3\ldots,x_p)=\frac{\frac{B(a_1+1,b_1-1)}{B(a_1,b_1)}+\frac{B(2,b_1-1)}{B(1,b_1)}\{\sum_{i=2}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1}-(p-1)\}}{(\sum_{i=2}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1})-(p-2)} \quad (4)$$

$$V(x_1/x_2,x_3\ldots,x_p)=E(x_1^2/x_2,x_3\ldots,x_p)-(E(x_1/x_2,x_3\ldots,x_p))^2 \quad (5)$$

Where $$E(x_1^2/x_2,x_3\ldots,x_p)=\frac{\frac{B(a_1+2,b_1-2)}{B(a_1,b_1)}+\frac{B(3,b_1-2)}{B(1,b_1)}\{\sum_{i=2}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1}-(p-1)\}}{(\sum_{i=2}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1})-(p-2)}$$

Proof: The n^{th} order moment of the distribution is

$$E(x_1^n/x_2,x_3\ldots,x_p)=\int_0^{\infty}x_1^n f(x_1/x_2,x_3\ldots,x_p)dx_1$$

$$E(x_1^n/x_2,x_3\ldots,x_p)=\int_0^{\infty}x_1^n\frac{\{(\sum_{i=1}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1})-(p-1)\}\frac{1}{B(1,b_1)(1+x_1)^{b_1+1}}}{(\sum_{i=2}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1})-(p-2)}dx_1$$

$$E(x_1^n/x_2,x_3\ldots,x_p)=\frac{\frac{B(a_1+n,b_1-n)}{B(a_1,b_1)}+\frac{B(n+1,b_1-n)}{B(1,b_1)}\{\sum_{i=2}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1}-(p-1)\}}{(\sum_{i=2}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1})-(p-2)}$$

If ***n = 1***, then the Conditional expectation is

$$E(x_1 / x_2, x_3 \ldots, x_p) = \frac{\frac{B(a_1+1, b_1-1)}{B(a_1,b_1)} + \frac{B(2,b_1-1)}{B(1,b_1)}\{\sum_{i=2}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1} - (p-1)\}}{(\sum_{i=2}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1}) - (p-2)}$$

If $\boldsymbol{n = 2}$, then the second order moment is

$$E(x_1^2 / x_2, x_3 \ldots, x_p) = \frac{\frac{B(a_1+2, b_1-2)}{B(a_1,b_1)} + \frac{B(3,b_1-2)}{B(1,b_1)}\{\sum_{i=2}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1} - (p-1)\}}{(\sum_{i=2}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1}) - (p-2)}$$

The conditional variance of the distribution is obtained by Substituting the first and second moments in (5).

Theorem 1.5: If there are $\boldsymbol{p = (q + k)}$ random variables, such that $\boldsymbol{q}$ random variables $X_1, X_2, X_3, .. X_q$ conditionally depends on the $\boldsymbol{k}$ variables $X_{q+1}, X_{q+2}, X_{q+3}, .. X_{q+k}$,then the density function of Sam-Solai's multivariate additive conditional Beta distribution is

$$f(x_1, x_2, x_3, \ldots x_q / x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{\{(\sum_{i=1}^{q+k}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1}) - (q+k-1)\}\prod_{i=1}^{q+k}\frac{1}{B(1,b_i)(1+x_i)^{b_i+1}}}{\{(\sum_{i=q+1}^{q+k}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1}) - (k-1)\}} \tag{6}$$

Where $0 \le x_i \le \infty$, $a_i, b_i > 0$

Proof: Let the multivariate conditional law for $\boldsymbol{q}$ random variables $X_1, X_2, X_3, .. X_q$ conditionally depending on the $\boldsymbol{k}$ variables $X_{q+1}, X_{q+2}, X_{q+3}, \ldots X_{q+k}$ is given as

$$f(x_1, x_2, x_3, \ldots x_q / x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{f(x_1, x_2, x_3, \ldots x_q, x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k})}{f(x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k})}$$

$$f(x_1, x_2, x_3, \ldots x_q / x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{\{(\sum_{i=1}^{q+k}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1}) - (q+k-1)\}\prod_{i=1}^{q+k}\frac{1}{B(1,b_i)(1+x_i)^{b_i+1}}}{\int_0^\infty\int_0^\infty\ldots\int_0^\infty\{(\sum_{i=1}^{q+k}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1}) - (q+k-1)\}\prod_{i=1}^{q+k}\frac{1}{B(1,b_i)(1+x_i)^{b_i+1}}\prod_{i=1}^{q}dx_i}$$

$$f(x_1, x_2, x_3, \ldots x_q / x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{\{(\sum_{i=1}^{q+k}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1}) - (q+k-1)\}\prod_{i=1}^{q+k}\frac{1}{B(1,b_i)(1+x_i)^{b_i+1}}}{\{(\sum_{i=q+1}^{q+k}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1}) - (k-1)\}}$$

Where $0 \le x_i \le \infty$, $a_i, b_i > 0$

Section 2: Constants of Sam-Solai's multivariate additive Beta distribution

Theorem 2.1: The Marginal product moments, Co-variance and Population Correlation Co-efficient between the Beta random variables X_1 and X_2 are given as

$$E(x_1 x_2) = \frac{a_1 + a_2 - 1}{(b_1 - 1)(b_2 - 1)} \tag{7}$$

$$COV(x_1, x_2) = \frac{a_1 + a_2 - (1 + a_1 a_2)}{(b_1 - 1)(b_2 - 1)} \tag{8}$$

$$\rho(x_1, x_2) = \frac{a_1 + a_2 - (1 + a_1 a_2)}{\sqrt{\frac{a_1(a_1 + b_1 - 1)a_2(a_2 + b_2 - 1)}{(b_1 - 2)(b_2 - 2)}}} \tag{9}$$

where $-1 \le \rho(x_1, x_2) \le +1$ for certain values of shape parameters (see Result3.4)

Proof: Assume that X_1 and X_2 are random variables from Sam-Solai's multivariate additive beta distribution. Let the product moment of the distribution is

$$E(x_1x_2)=\int_0^\infty\int_0^\infty\cdots\int_0^\infty x_1x_2 f(x_1,x_2,x_3,\ldots x_p)\prod_{i=1}^p dx_i$$

Its Co-variance is $COV(x_1,x_2)=E(x_1x_2)-E(x_1)E(x_2)$ (10)

Then

$$E(x_1x_2)=\int_0^\infty\int_0^\infty\cdots\int_0^\infty x_1x_2\{(\sum_{i=1}^p\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1})-(p-1)\}\prod_{i=1}^p\frac{1}{B(1,b_i)(1+x_i)^{b_i+1}}\prod_{i=1}^p dx_i$$

By evaluation, it follows that $E(x_1x_2)=\frac{a_1+a_2-1}{(b_1-1)(b_2-1)}$. The marginal expectation of beta random variables x_1 and x_2 is (a_1/b_1-1) and (a_2/b_2-1) respectively. The marginal Product moment for $E(x_1x_2)$ is obtained by substituting the above marginal expectations for x_1 and x_1 in (10).

Thus

$$COV(x_1,x_2)=\frac{a_1+a_2-(1+a_1a_2)}{(b_1-1)(b_2-1)} \quad (11)$$

Correlation coefficient of a distribution is (12a)

$$\rho(x_1,x_2)=\frac{COV(x_1,x_2)}{\sigma_1\sigma_2}$$

It observes that $\sigma_1=\sqrt{a_1(a_1+b_1-1)/(b_1-1)^2(b_1-2)}$ and $\sigma_2=\sqrt{a_2(a_2+b_2-1)/(b_2-1)^2(b_2-2)}$ (12b)

From (11), (12a) and (12b), it follows that

$$\rho(x_1,x_2)=\frac{a_1+a_2-(1+a_1a_2)}{\sqrt{\frac{a_1(a_1+b_1-1)a_2(a_2+b_2-1)}{(b_1-2)(b_2-2)}}} \quad (13)$$

Where $-1\le\rho(x_1,x_2)\le+1$ for certain values of shape parameters

Remark 2.1*:* The Product moments, Co-variance and population Correlation Co-efficient between the i^{th} and j^{th} Beta random variables are given as

$$E(x_ix_j)=\frac{a_i+a_j-1}{(b_i-1)(b_j-1)} \quad (14)$$

$$COV(x_i,x_j)=\frac{a_i+a_j-(1+a_ia_j)}{(b_i-1)(b_j-1)} \quad (15)$$

$$\rho(x_i,x_j)=\frac{a_i+a_j-(1+a_ia_j)}{\sqrt{\frac{a_i(a_i+b_i-1)a_j(a_j+b_j-1)}{(b_i-2)(b_j-2)}}} \quad (16)$$

Where, $i\ne j, -1\le\rho(x_1,x_2)\le+1$ for certain values of shape parameters

Theorem 2.2: The Moment generating function of Sam-Solai's Multivariate additive Beta distribution is

$$M_{x_1,x_2\ldots x_p}(t_1,t_2\ldots t_p)=\prod_{i=1}^p\frac{\sum_{k=0}^\infty\frac{t_i^kB(1+k,b_i-k)}{k!}}{B(1,b_i)}\{\sum_{i=1}^p(\frac{B(1,b_i)\sum_{k=0}^\infty\frac{t_i^kB(a_i+k,b_i-k)}{k!}}{B(a_i,b_i)\sum_{k=0}^\infty\frac{t_i^kB(1+k,b_i-k)}{k!}}-(p-1)\} \quad (17)$$

Proof: Let the moment generating function of a Multivariate distribution is given as

$$M_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_3,\ldots t_p)=\int_0^\infty\int_0^\infty\cdots\int_0^\infty e^{\sum_{i=1}^p t_ix_i} f(x_1,x_2,x_3,\ldots x_p)\prod_{i=1}^p dx_i$$

$$M_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_3,\ldots t_p)=\int_0^\infty\int_0^\infty\cdots\int_0^\infty e^{\sum_{i=1}^p t_ix_i}\{(\sum_{i=1}^p\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1})-(p-1)\}\prod_{i=1}^p\frac{1}{B(1,b_i)(1+x_i)^{b_i+1}}\prod_{i=1}^p dx_i$$

$$M_{x_1,x_2\dots x_p}(t_1,t_2\dots t_p)=\prod_{i=1}^{p}\frac{\sum_{k=0}^{\infty}\frac{t_i^{k}B(1+k,b_i-k)}{k!}}{B(1,b_i)}\{\sum_{i=1}^{p}(\frac{B(1,b_i)\sum_{k=0}^{\infty}\frac{t_i^{k}B(a_i+k,b_i-k)}{k!}}{B(a_i,b_i)\sum_{k=0}^{\infty}\frac{t_i^{k}B(1+k,b_i-k)}{k!}}-(p-1)\}$$ by integrating the above equation.

Theorem 2.3: The Cumulant of the Moment generating function of the Sam-Solai's Multivariate additive Beta distribution is

$$C_{x_1,x_2\dots x_p}(t_1,t_2\dots t_p)=\sum_{i=1}^{p}\log(\sum_{k=0}^{\infty}\frac{t_i^{k}B(1+k,b_i-k)}{k!})-\sum_{i=1}^{p}\log(B(1,b_i))+\log(\sum_{i=1}^{p}(\frac{B(1,b_i)\sum_{k=0}^{\infty}\frac{t_i^{k}B(a_i+k,b_i-k)}{k!}}{B(a_i,b_i)\sum_{k=0}^{\infty}\frac{t_i^{k}B(1+k,b_i-k)}{k!}}-(p-1)) \quad (18)$$

Proof: It is found from $C_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_3,\dots t_p)=\log(M_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_3,\dots t_p))$

2.4: The Characteristic function of the Sam-Solai's Multivariate additive Beta distribution is

$$\phi_{x_1,x_2\dots x_p}(t_1,t_2\dots t_p)=\prod_{j=1}^{p}\frac{\sum_{k=0}^{\infty}\frac{(it_j)^{k}B(1+k,b_j-k)}{k!}}{B(1,b_j)}\{\sum_{j=1}^{p}(\frac{B(1,b_j)\sum_{k=0}^{\infty}\frac{(it_j)^{k}B(a_i+k,b_j-k)}{k!}}{B(a_j,b_j)\sum_{k=0}^{\infty}\frac{(it_j)^{k}B(1+k,b_j-k)}{k!}}-(p-1)\} \quad (19)$$

Proof: Let the characteristic function of a multivariate distribution is given as

$$\phi_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_3,\dots t_p)=\int_0^{\infty}\int_0^{\infty}\dots\int_0^{\infty}e^{i\sum_{j=1}^{p}t_jx_j}f(x_1,x_2,x_3,\dots x_p)\prod_{j=1}^{p}dx_j$$

$$\phi_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_3,\dots t_p)=\int_0^{\infty}\int_0^{\infty}\dots\int_0^{\infty}e^{i\sum_{j=1}^{p}t_jx_j}\{(\sum_{j=1}^{p}\frac{B(1,b_j)}{B(a_j,b_j)}(\frac{x_j}{1+x_j})^{a_j-1})-(p-1)\}\prod_{j=1}^{p}\frac{1}{B(1,b_j)(1+x_j)^{b_j+1}}\prod_{j=1}^{p}dx_j$$

$$\phi_{x_1,x_2\dots x_p}(t_1,t_2\dots t_p)=\prod_{j=1}^{p}\frac{\sum_{k=0}^{\infty}\frac{(it_j)^{k}B(1+k,b_j-k)}{k!}}{B(1,b_j)}\{\sum_{j=1}^{p}(\frac{B(1,b_j)\sum_{k=0}^{\infty}\frac{(it_j)^{k}B(a_i+k,b_j-k)}{k!}}{B(a_j,b_j)\sum_{k=0}^{\infty}\frac{(it_j)^{k}B(1+k,b_j-k)}{k!}}-(p-1)\}$$ by integrating the above equation.

Theorem 2.5: The survival function of the Sam-Solai's Multivariate additive Beta distribution is

$$S(x_1,x_2,x_3\dots,x_p)=1-\{(\sum_{i=1}^{p}\frac{(1+x_i)^{b_i}\phi_i(x_i;a_i,b_i)}{((1+x_i)^{b_i}-1)})-(p-1)\}\prod_{i=1}^{p}\frac{((1+x_i)^{b_i}-1)}{(1+x_i)^{b_i}} \quad (20)$$

Proof: Let the survival function of a multivariate distribution is given as

$$S(x_1,x_2,x_3\dots,x_p)=1-F(x_1,x_2,x_3\dots,x_p)$$

$$S(x_1,x_2,x_3\dots,x_p)=1-\int_0^{x_1}\int_0^{x_2}\int_0^{x_3}\dots\int_0^{x_p}\{(\sum_{i=1}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{u_i}{1+u_i})^{a_i-1})-(p-1)\}\prod_{i=1}^{p}\frac{1}{B(1,b_i)(1+u_i)^{b_i+1}}du_i$$

$$S(x_1,x_2,x_3\dots,x_p)=1-\{(\sum_{i=1}^{p}\frac{B(1,b_i)b_i(1+x_i)^{b_i}}{((1+x_i)^{b_i}-1)}\int_0^{x_i}\frac{u_i^{a_i-1}}{B(a_i,b_i)(1+u_i)^{a_i+b_i}}du_i)-(p-1)\}\prod_{i=1}^{p}\frac{((1+x_i)^{b_i}-1)}{B(1,b_i)b_i(1+x_i)^{b_i}}$$

$$S(x_1,x_2,x_3\dots,x_p)=1-\{(\sum_{i=1}^{p}\frac{(1+x_i)^{b_i}\phi_i(x_i;a_i,b_i)}{((1+x_i)^{b_i}-1)})-(p-1)\}\prod_{i=1}^{p}\frac{((1+x_i)^{b_i}-1)}{(1+x_i)^{b_i}}$$

where $\phi_i(x_i;a_i,b_i)=\int_0^{x_i}\frac{u_i^{a_i-1}}{B(a_i,b_i)(1+u_i)^{a_i+b_i}}du_i$ is the lower incomplete Beta integral of i^{th} random variable.

Theorem 2.6: The hazard function of the Sam-Solai's Multivariate additive Beta distribution is

$$h(x_1,x_2,x_3\dots,x_p)=\frac{\{(\sum_{i=1}^{p}\frac{B(1,b_i)}{B(a_i,b_i)}(\frac{x_i}{1+x_i})^{a_i-1})-(p-1)\}\prod_{i=1}^{p}\frac{1}{B(1,b_i)(1+x_i)^{b_i+1}}}{1-\{(\sum_{i=1}^{p}\frac{(1+x_i)^{b_i}\phi_i(x_i;a_i,b_i)}{((1+x_i)^{b_i}-1)})-(p-1)\}\prod_{i=1}^{p}\frac{((1+x_i)^{b_i}-1)}{(1+x_i)^{b_i}}} \quad (21)$$

Proof: It is obtained from

$$h(x_1,x_2,x_3\dots,x_p)=\frac{f(x_1,x_2,x_3\dots,x_p)}{S(x_1,x_2,x_3\dots,x_p)} \text{ and } S(x_1,x_2,x_3\dots,x_p)=1-F(x_1,x_2,x_3\dots,x_p)$$

Theorem 2.7: The Cumulative hazard function of the Sam-Solai's Multivariate additive Beta distribution is

$$H(x_1,x_2,x_3\dots,x_p)=-\log(1-\{(\sum_{i=1}^{p}\frac{(1+x_i)^{b_i}\phi_i(x_i;a_i,b_i)}{((1+x_i)^{b_i}-1)})-(p-1)\}\prod_{i=1}^{p}\frac{((1+x_i)^{b_i}-1)}{(1+x_i)^{b_i}}) \quad (22)$$

Proof: Let the Cumulative hazard function of a multivariate distribution is given as

$$H(x_1,x_2,x_3\dots,x_p)=-\log(1-F(x_1,x_2,x_3\dots,x_p)$$

$$H(x_1,x_2,x_3\dots,x_p)=-\log(S(x_1,x_2,x_3\dots,x_p))$$

$$H(x_1,x_2,x_3\dots,x_p)=-\log(1-\{(\sum_{i=1}^{p}\frac{(1+x_i)^{b_i}\phi_i(x_i;a_i,b_i)}{((1+x_i)^{b_i}-1)})-(p-1)\}\prod_{i=1}^{p}\frac{((1+x_i)^{b_i}-1)}{(1+x_i)^{b_i}})$$

Section 3: Some Special Cases

Results 3.1: The uni-variate marginal of the Sam-Solai's multivariate additive Beta distribution of kind-2 of Type-A are the uni-variate two parameter Beta distributions of second Kind.

Result 3.2*:* If ***P*=1**, the Sam-Solai's multivariate additive Beta density is reduced to density of univariate two parameter Beta distribution of second Kind.

Result 3.3: If ***P*=2**, then the density of Sam-Solai's Multivariate Beta distribution of second kind was reduced into

$$f(x_1,x_2)=(\frac{B(1,b_1)}{B(a_1,b_1)}(\frac{x_1}{1+x_1})^{a_1-1}+\frac{B(1,b_2)}{B(a_2,b_2)}(\frac{x_2}{1+x_2})^{a_2-1}-1)\frac{1}{B(1,b_1)(1+x_1)^{b_1+1}B(1,b_2)(1+x_2)^{b_2+1}} \quad (23)$$

where $0\le x_1,x_2\le\infty,\ a_1,a_2,b_1,b_2>0$

This is called Sam-Solai's Bi-variate additive Beta distribution of Kind-2 of Type-A.

Result 3.4: The table 1, 2 and Bi-variate probability surface for (23) shows the selected simulated standard Bi-variate correlations between two Beta random variables which are bounded between -1 and +1 calculated from 10,000 different combinations of shape parameter a_1,a_2,b_1,b_2.

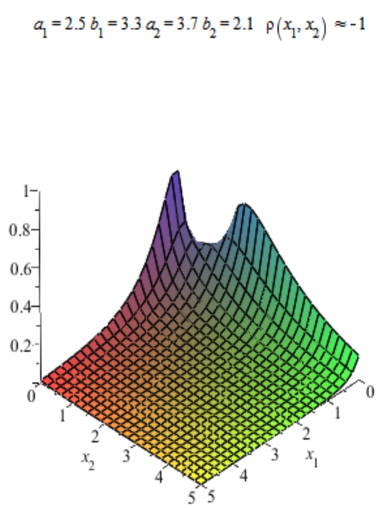

Figure-1

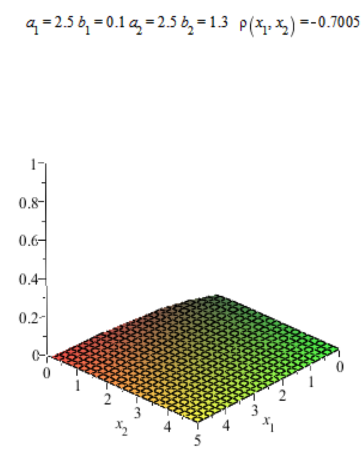

Figure-2

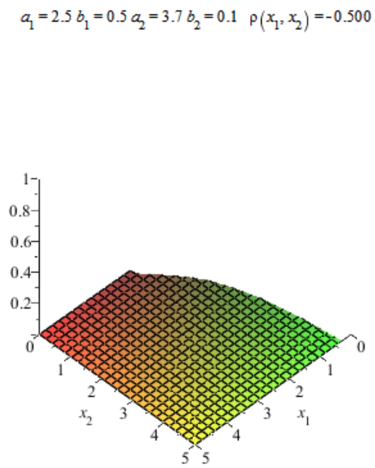

Figure-3

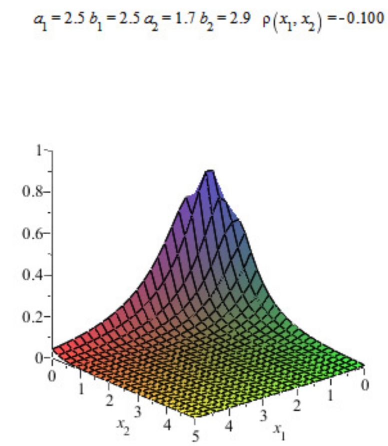

Figure-4

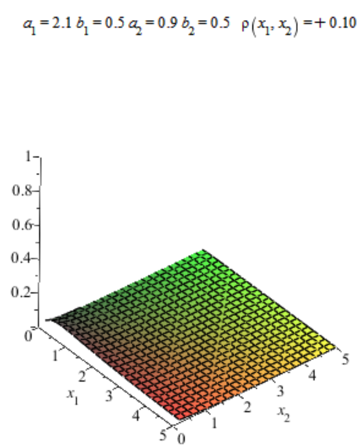

Figure-5

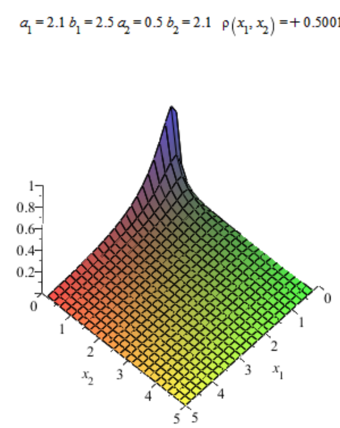

Figure-6

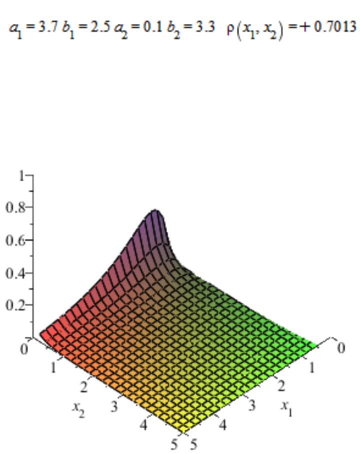

Figure-7

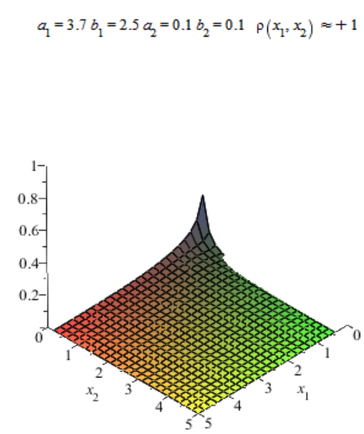

Figure-8

Result:3.5 From(1) and If $y_i = x_i / 1 + x_i$, then the density of Sam-Solai's Multivariate additive Beta distribution of Kind-2 of Type-A transformed into Sam-Solai's Multivariate additive Beta distribution of Kind-1 of Type-A and its density function is given as

$$f(y_1, y_2, y_3 \dots, y_p) = \{(\sum_{i=1}^{p} \frac{y_i^{a_i - 1} B(1, b_i)}{B(a_i, b_i)}) - (p-1)\} \prod_{i=1}^{p} \frac{(1 - y_i)^{b_i - 1}}{B(1, b_i)} \quad (24)$$

Where $0 \le y_i < 1 \quad a_i, b_i > 0$

Result:3.6- From (1) and If $a_i = m_i / 2, b_i = n_i / 2$ and $y_i = n_i x_i / m_i$,then the Sam-solai's Multivariate additive Beta distribution of Kind-2 of Type-A transformed into Sam-solai's Multivariate additive F-distribution of Kind-1 with (m_i, n_i) degrees of freedom and its density function is given as

$$f(y_1, y_2, y_3 \dots, y_p) = \{(\sum_{i=1}^{p} \frac{B(1, \frac{n_i}{2})}{B(\frac{m_i}{2}, \frac{n_i}{2})} (\frac{(m_i / n_i) y_i}{1 + (m_i / n_i) y_i})^{\frac{m_i}{2} - 1}) - (p-1)\} \prod_{i=1}^{p} (\frac{(m_i / n_i)}{B(1, \frac{n_i}{2})})(\frac{1}{1 + (m_i / n_i) y_i})^{\frac{n_i}{2} + 1} \quad (25)$$

Where $0 \le y_i \le \infty \quad m_i, n_i > 0$

Result:3.7- From (1) and If $y_i = -\log x_i$, then the Sam-solai's Multivariate additive Beta distribution of Kind-2 of Type-A transformed into Sam-solai's generalization of Multivariate standard Logistic- Beta distribution of Kind-1 with parameters (a_i, b_i) and its density function is given as

$$f(y_1, y_2, y_3 \dots, y_p) = \{(\sum_{i=1}^{p} \frac{B(1, b_i)}{B(a_i, b_i)} (\frac{e^{-y_i}}{1 + e^{-y_i}})^{a_i - 1}) - (p-1)\} \prod_{i=1}^{p} \frac{e^{-y_i}}{B(1, b_i)(1 + e^{-y_i})^{b_i + 1}} \quad (26)$$

Where $-\infty \le y_i \le +\infty$, $a_i, b_i > 0$

Result:3.8-From(1) and If $a_i = b_i = 1, y_i = -\log x_i$,then the Sam-solai's Multivariate additive Beta distribution of Kind-2 of Type-A changed into product of univariate standard Logistic distribution and it is given as

$$f(y_1, y_2, y_3 \dots, y_p) = \prod_{i=1}^{p} \frac{e^{-y_i}}{(1 + e^{-y_i})^2} \quad (27)$$

where $-\infty \le y_i < +\infty$

Conclusion

The multivariate generalization of two parameter Beta distribution in an additive form of Sam-Solai's generalization having some interesting features. At first, the marginal uni-variate distributions of the Sam-Solai's Multivariate additive Beta distribution are uni-variate and enjoyed the symmetric property. Secondly, the Population Correlation co-efficient of the proposed distribution is bounded between -1 and +1 for certain values of shape parameter and the authors established the simulated standard bivariate correlations. Finally, the multivariate generalization of two parameter Beta distribution in an additive form open the way for the same additive form of the transformation of Sam-Solai's Multivariate additive Beta distribution of kind-2 of Type-A into Multivariate additive Beta distribution of Kind-1 of Type-A, Multivariate F-distribution of Kind-1,Multivariate standard Logistic-Beta distribution of Kind-1.

References

1. Srivastava M.S., on the distribution of a multiple correlation matrix, non-central Multivariate beta distributions, Ann. *Math. Statist*., **39**, 277-232 **(1968)**
2. W. Y. Tan**,** Note on the multivariate and the generalized multivariate beta distributions, *J.Amer. Statist. Assoc*. **64**, 230–241(**1969)**
3. Khatri C.G.**,** A note on Mitra's paper "A density-free approach to the matrix variate beta distribution", *Sankhya Ser*., **A 32**, 311–318 **(1970)**
4. Javier W.R. and Gupta A.K., On generalized matric variate beta distributions, *Statistics,* **16(4)**, 549–558 **(1985)**
5. Konno Y., Exact moments of the multivariate F and beta distributions, *J. Japan Statist*. Soc., **18(2)**, 123–130 **(1988)**

6. Khattree R. and Gupta R.D., some probability distributions connected with beta and Gamma matrices, *Comm. Statist., Theory Methods,* **21(2)**, 369–390 **(1992)**

7. Uhlig H., On singular Wishart and singular multivartiate beta distributions, *Ann.Statistic*., **22(15),** 395-405 **(1994)**

8. McDonald J.B. and Yexiao J. Xu, A generalization of the beta distribution with applications, *Journal of Econometrics*, **66**, 133–152 **(1995)**

9. Gupta K. and Nagar D.K., Matrix variate beta distribution, *Int. J.Maths and Math. Sci*., **23(7)**, 449–459 **(2000)**

10. Jones, M.C. Multivariate t and beta distributions associated with the multivariate F-distribution, *Metrika*, **54**, 215-231 **(2001)**

11. Nagar D.K. and Gupta A.K.**,** Matrix variate Kummer-Beta distribution, *Journal of Australian Mathematical Society*, **73(1)**, 11–25 **(2002)**

12. Gupta K. and Nadarajah S. (Eds.), Handbook of Beta Distribution and Its Applications, New York, Marcel Dekker **(2004)**

13. Nadarajah S. and Kotz S., Some beta distributions, *Bulletin of the Brazilian Mathematical Society*, New Series, **37(1)**, 103–125 **(2006)**

14. D´ıaz-Garc´ıa J.A. and Guti´errez-J´aimez R., Non-central, nonsingular matrix variate beta distribution, *Brazilian J. Prob. Statist.* **21**, 175–186 **(2007)**

Table-1
Simulation Runs for Selected Values of Shape Parameter with Population Correlation Bounds

Runs	a_1	b_1	a_2	b_2	$\rho(x_1, x_2)$
3085	2.50	3.30	3.70	2.10	≈ -1.00
8927	2.90	0.10	2.10	1.70	-0.9007
2392	1.70	0.10	2.50	1.70	-0.8011
3091	2.50	0.10	2.50	1.30	-0.7005
5648	2.90	1.30	3.30	1.70	-0.6031
6201	2.50	0.50	3.70	0.10	-0.5000
2338	2.90	0.50	1.70	1.30	-0.4002
5667	2.50	1.30	1.70	1.70	-0.3001
2612	3.30	2.90	2.50	3.30	-0.2000
5098	2.50	2.50	1.70	2.90	-0.1000
638	2.10	0.50	0.90	0.50	+0.1000
4624	0.90	0.50	2.10	0.50	+0.1000
4091	0.50	2.10	2.10	2.50	+0.1000
2770	0.50	2.50	2.10	2.50	+0.2000
6502	2.10	2.50	0.50	2.50	+0.2000
7080	0.50	1.30	1.70	1.30	+0.3001
8270	1.70	1.30	0.50	1.30	+0.3001
8126	2.50	3.30	0.50	0.10	+0.4005
3336	2.10	2.50	0.50	2.10	+0.5001
272	0.50	0.90	2.10	0.90	+0.6001
5061	2.10	0.90	0.50	0.90	+0.6001
7366	2.10	2.50	0.10	2.90	+0.6001
5614	3.30	2.10	0.10	0.50	+0.7013
7720	3.70	2.50	0.10	3.30	+0.7013
4991	0.10	3.70	2.90	3.70	+0.8019
8378	2.90	3.70	0.10	3.70	+0.8019
2502	0.10	2.50	2.10	2.50	+0.9001
2861	2.10	2.50	0.10	2.50	+0.9001
890	3.70	2.50	0.10	0.10	≈ 1.00

Table-2
Simulation Runs for Selected Values of Shape Parameter with Population Correlation Bounds When $\rho(x_1, x_2) \approx 0$

Runs	a_1	b_1	a_2	b_2
2526	1.30	2.10	.90	3.70
9253	.90	2.10	1.30	3.70
7683	1.30	2.10	.90	3.30
9132	.90	2.10	1.30	3.30
4151	1.30	2.10	.90	2.90
108	.90	2.10	1.30	2.90
7979	1.70	2.10	.90	3.70
2923	.90	2.10	1.70	3.70
549	1.30	2.50	.90	3.70
5086	.90	2.50	1.30	3.70
388	1.30	2.10	.90	2.50
9107	.90	2.10	1.30	2.50
2031	1.70	2.10	.90	3.30
5010	.90	2.10	1.70	3.30
2589	2.10	2.10	.90	3.70
4671	1.30	2.50	.90	3.30
9841	.90	2.50	1.30	3.30
3235	1.30	2.90	.90	3.70
1122	.90	2.90	1.30	3.70
6434	.90	2.10	2.10	3.70
129	2.50	2.10	.90	3.70
8866	1.30	3.30	.90	3.70
8095	1.70	2.10	.90	2.90
3718	.90	3.30	1.30	3.70
7387	2.10	2.10	.90	3.30
4774	.90	2.10	1.70	2.90
722	1.30	2.90	.90	3.30
8453	.90	2.90	1.30	3.30
3096	1.30	3.70	.90	3.70
3704	.90	3.70	1.30	3.70
6698	1.30	2.50	.90	2.90
209	.90	2.50	1.30	2.90
8869	.90	2.10	2.10	3.30
5632	2.90	2.10	.90	3.70
7483	.90	2.10	2.50	3.70
3170	3.30	2.10	.90	3.70
8560	2.50	2.10	.90	3.30
7624	.90	3.30	1.30	3.30
9048	1.30	3.30	.90	3.30
5762	.90	2.10	2.90	3.70
8989	3.70	2.10	.90	3.70
8083	.90	3.70	1.30	3.30
4579	1.30	3.70	.90	3.30
4756	2.10	2.10	.90	2.90
1712	.90	2.10	2.50	3.30
3110	1.70	2.50	.90	3.70
1027	2.90	2.10	.90	3.30
9831	.90	2.90	1.30	2.90
7622	1.30	2.90	.90	2.90
4519	.90	2.10	3.30	3.70
4981	.90	2.10	2.10	2.90
4897	.90	2.50	1.70	3.70
2354	1.70	2.10	.90	2.50