Research Journal of Engineering Sciences ISSN 2278 – 9472
Vol. **6(8),** 24-28, September **(2017)** *Res. J. Engineering Sci.*

Short Review Paper

Time varying delay systems: a survey

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Available online at: www.isca.in, www.isca.me
Received 8th April 2017, revised 8th September 2017, accepted 22th September 2017

Abstract

The development of the hardware systems has incurred various types of delays such as processing and transmission delays. Such delay may be due to the effect of tolerances of electronic components which were used while developing the system. Such time delay parameters must be implemented in the transfer function of the system so as to identify the correct cause of dynamic behavior of the system which in turn affects the stability of the system. For the development of the accurate system it is required to consider the condition for the global robust asymptotic stability. Criteria for verifying robust stability are formulated as feasibility problems over a set of frequency dependent linear matrix inequalities. The criteria can be equivalently formulated as Semi-Definite Programs (SDP) using Kalman-Yakubovich-Popov lemma. Therefore, checking robust stability can be performed in a computationally efficient fashion. The Lyapunov-Krasovskii approach is definitely the most popular method to address this issue and many results have proposed new functionals and enhanced techniques for deriving less conservative stability conditions. The paper surveys the techniques used for developing the stable system from the various literatures published recently and draws the result that which method is best to develop a reliable and stable systems.

Keywords: Semi-Definite Programs (SDP), Stability, Time Varying System, Integral Quadratic Constraint.

Introduction

It is well-known that the manifestation of time delays in a system can lead to performance degradation and even destabilization of the system. For systems in which the signal transmission time delays among sensors, compensators and actuators are small, compared to the time constant of the overall system, the effect of time delay is often not significant enough to cause serious problems. This is not the case, however, within the context of large-scale distributed and networked systems, where the effects of time delays can be very significant. Time delay system is a subclass of infinite dimensional systems that has been frequently employed since it can model commonly arising transport and propagation phenomena. For such systems, time-delay robustness must be explicitly addressed to ensure that system-level performance is achieved in a robust manner. Time-delay robustness is often studied for situations in which the delay is uncertain but remains constant throughout time. Delays can be encountered in many processes such as biology, chemistry, economics, and population dynamics[1] as well as in networks[2]. However, delays are the origin of performance and stability degradation, which thus have motivated a lot of work. While much research has been done and stability criteria have been derived for systems with uncertain constant time-delays, the recent emphasis has been put on the scenario where the time delay is time varying. The significance of such problems is tied to the recent ample interest in designing control algorithms for large-scale networked systems. For applications within the scope of large-scale distributed and networked systems, such as the regulation of internet traffic and control over networked communication channels. Other engineering applications where time-varying delays appear include real-time implementation of control systems and control of fuel injection systems. The results obtained while the research work by authors have been extended to time varying delay systems either using adapted Lyapunov-Krasovskii[3-11] or robustness tools[12-14]. These latter methodologies often require, explicitly or implicitly, the delay-free system to be stable, which is a rather important restriction. Time-varying delay systems are expressed in terms of Linear Matrix Inequalities (LMIs) which may be solved efficiently with Semi-Definite Programming (SDP).

Preliminaries

This section presents the function and variation characteristic of the system.

Consider the following time-varying delay system:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-h(t)) & \forall t \geq 0, \\ x(t) = \phi(t) & \forall t \in [-h_{\max}, 0], \end{cases} \quad (1)$$

Where: x(t) $\in R_n$ is the state vector, - is the initial condition and A, $A_d \in R_{n\times n}$ are constant matrices. The delay h is time varying with

$h(t) \in [h_{min}, h_{max}]$ and $|\dot{h}(t)| \leq d$, (2)

Where, h_{min}, h_{max} and d are given positive constants. In this work, we aim at assessing the stability of system (1) via different stability principle developed for robust control. We will show that various criteria, related to the available information on the delay, can be derived choosing appropriately a set of operators.

Related work

This section presents the related work previously produced by the researchers. Basically the methodologies for achieving the stability and controllability have been discussed below.

Jin-Hoon Kim[15] presented the stability of linear systems with a time-varying delay using Lyapunov–Krasovskii functional (LKF) approach. The authors main contribution to this paper is to present a result expressed in the form of LMI which overcomes the upper bound $\max_{\mu \to 0} h \leq 4.4721$ using a new simple LKF. The basic ideas in constructing a new LKF are adoption of the cross terms of variables and quadratic terms multiplied by a higher degree scalar function. In recent works[16], the property of first order convex combination property was used. Similarly, in this paper, the property of quadratic convex function, namely Lemma 1, is used. Also, Lemma 2 gives an upper bound of the integral of quadratic function multiplied by a $1^{st}/2^{nd}$ degree scalar function. The author concluded that by considered the stability of time-delayed linear systems with a time-varying delay in the case of simple LKF approach without delay decomposition, the allowable maximum size of delay had a fixed bound for years. To overcome this, the author constructed a new simple LKF which has the cross terms of variables and quadratic terms multiplied by a higher degree scalar function. And then, using the property of a quadratic convex function and an upper bound of the integral of quadratic multiplied by a scalar function, author derived a delay-dependent stability criterion in the form of LMIs. Finally, by two well-known examples, author presented the usefulness of the result.

Yassine Ariba et al.[17] presented an original approach: the quadratic separation. For this the author exploited the delay operator properties to provide delay range stability conditions. In particular, L2-norm of delay-dependent operators is computed so as to reduce the conservatism of the approach.

Moreover, the main result is able to assess the stability of non-small delay systems, i.e., it can detect a stability interval for systems that are unstable without any delay. The quadratic separation provides a fruitful framework to address stability of non-linear and uncertain systems[18,19]. Recent studies[20] have shown that such a framework reduces significantly the conservatism of the stability analysis of time-delay systems with constant delay. In this paper authors extend this method to time varying delay systems, which involves the development of results for a new set of operators. This result includes two conditions: a matrix inequality related to the lower block of the feedback system and an inner product that states an integral quadratic constraint (IQC) on the upper block. The author concluded that using an augmented state, which emphasizes the relation between $\dot{h}$ and $(\dot{x}, \ddot{x})$, the resulting criteria have been expressed in terms of a convex optimization problem with LMI constraints. Finally, numerical examples showed that this method reduced conservatism and improve the maximal allowable interval on the delay.

E. Fridman et al.[21] says that in the researched work cited, descriptor model transformation has been introduced. The authors compare methods under different transformations and show the advantages of the descriptor one. Authors also obtained new delay-dependent stability conditions for systems with time varying delays in terms of linear matrix inequalities. Authors also refine results on delay-dependent H_∞ control and extend them to the case of time-varying delays. Numerical examples illustrated the effectiveness of the method. Previously two main approaches for dealing with the stability of systems with time-varying delays was available. The first is based on Lyapunov–Krasovskii functionals and the second is based on Razumikhin theory. The authors considered two main cases of time-varying delays: i. differentiable uniformly bounded delays with delay-derivatives bounded by d < 1; and ii. continuous uniformly bounded delays.

Author says to the best of their knowledge, the Razumikhin approach was the only one that was able to cope with the second case, which allows fast time-varying delays. In his paper the author shed more light on the conservatism of the various model transformations and shows the advantages of the descriptor one. Authors revealed the sources for the conservatism of delay-dependent stability methods and the advantages of the one under descriptor transformation have been demonstrated. The maximum values of h that still allow stability via state-feedback are depicted in Figure-1 as a function of d.

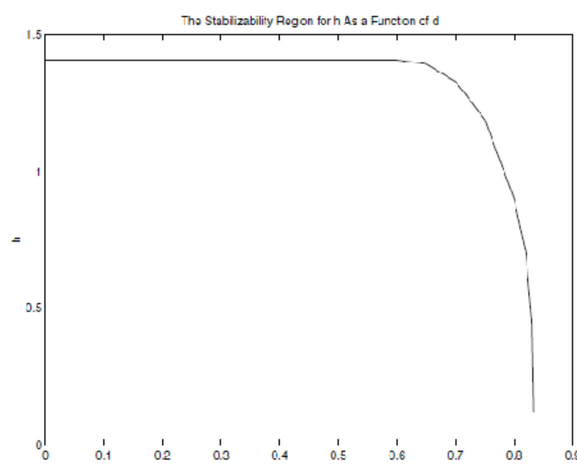

Figure-1: The stabilizability bounds for the time-delay h as a function of d.

In Figure-2 authors describe the minimum achievable value of γ as a function of d for h = 1.38 and for $\varepsilon_1 = -0.29$ and $\varepsilon_2 = -1$. The latter value of h is quite close to the maximum achievable value of h = 1.408.

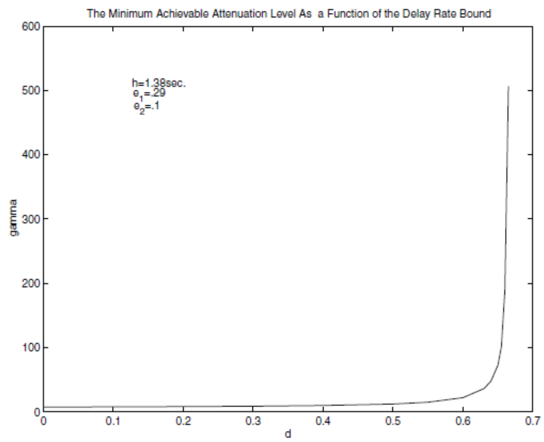

Figure-2: The minimum achievable attenuation level as a function of d for h =1:38.

Two types of results for systems with time-varying delays have been derived: delay-dependent/rate-dependent and delay-dependent/rate-independent. In both cases, the new stability results are less restrictive than the existing results[15] obtained for the first (less robust) case. The authors results for the second case, which includes fast-varying delays, seem to be less conservative than those of Fridman and Shaked[21].

Stability of discrete time system with time varying delay

Based on the summation inequalities, the following stability theorem is provided.

Assume that there exist matrices P in S_+^{2n}, Q_1, Q_2, Z_1, Z_2, in S_+^n and a matrix X in $\square^{2n\times 2n}$ such that

$$\psi \succ 0, \psi(h1) \prec 0, \psi(h1) \prec 0, \quad (3)$$

Where

$$\phi(h) = F_1^T \left(P + \begin{bmatrix} h_1^2 Z_1 + h_{12}^2 Z_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) F_1$$

$$-F_1^T P F_2 + He(\Gamma^T(h) P F_{12}) + \hat{Q} - \Pi^T \Psi \Pi,$$

$$\Psi = \begin{bmatrix} \tilde{Z}_1(h_1) & 0 & 0 \\ 0 & \tilde{Z}_2 & X \\ 0 & X^T & \tilde{Z}_2 \end{bmatrix},$$

$$\hat{Q} = diag(Q_1, -Q_1 + Q_2, 0, -Q_2, 0, 0, 0)$$

$$\tilde{Z}_1(h_1) = diag(Z_1 \gamma(h_1) Z_1), \tilde{Z}_2 = diag(Z_2 = (Z_2, 3Z_2)$$

$$F_1 = \begin{bmatrix} A-I & 0 & A_d & 0 & 0 & 0 & 0 \\ 0 & -I & 0 & 0 & I & 0 & 0 \\ 0 & 0 & -I & -I & 0 & I & I \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -I & 0 & 0 & 0 & I & 0 & 0 \\ 0 & -I & -I & 0 & 0 & I & I \end{bmatrix}$$

$$\Gamma(h) = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_1 I & 0 & 0 \\ 0 & -I & -I & 0 & 0 & (h-h_1)I & (h_2-h)I \end{bmatrix}$$

$$\Pi = \begin{bmatrix} M & & 0_{2n\times 2n} \\ 0_{2n\times n} & M & 0_{2n\times n} \\ 0_{2n\times 2n} & & M \end{bmatrix}$$

$$M = \begin{bmatrix} I & -I & 0 & 0 & 0 \\ I & I & 0 & 0 & -2I \end{bmatrix}$$

and where $h_{12} = h_2 - h_1, F_{12} = F_1 - F_2, \gamma(h_1) = 1$ if $h_1 = 1$

$\gamma(h_1) = (h_1+1)/(h_1-1) > 1$ if $h_1 > 1$. Then system (1) is asymptotically stable for any time-varying delay $h(k) \in [h_1, h_2]$.

Results

The system presented above is simulated in Matlab 2014a version and the outputs are obtained. The values of h_1 and h_2 have been changed the test of stability has been performed as shown below:

Table-1: Admissible upper bound h2 for various h1, for the system.

S. No.	h1	h2
1.	1	20
2.	3	21
3.	5	21
4.	7	22
5.	9	23
6.	11	24

When the delay is constant and $h1 = h2 = 1$ and $h1 = h2 = 5$, aneigen value analysis shows that the maximal acceptable constant sampling period $T_m = T_M = T$ for which the system remains asymptotically stable is T = 0.76 and T = 0.21, respectively. It has been also noted that the summation inequality is not conservative when $h_1 = h_2 = 1$. Hence, the conservatism in this case comes from the approximation the uncertain system.

Conclusion

In this technical note the summation inequality presented is relevant for the stability analysis of discrete time systems with interval time-varying delays.

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