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Analysis of Flexural Members using an Alternative approach

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Abstract

An alternative approach used for the analysis of flexural members is method of initial functions (MIF). The equations of two dimensional elasticity have been used for deriving the governing equations. Numerical solutions of the governing equations have been presented for simply supported orthotropic beam. The method of initial function (MIF) is an analytical method of elasticity theory. The method makes it possible to obtain exact solutions of different types of problems, i.e., solutions without the use of hypotheses about the character of stress and strain. This method has applications in various fields of structural engineering such as plates, shells and beams. It is very useful in case of thick, sandwich, and layered beams.

Keywords: Flexural members, method of initial functions, stress, strain, flexural Member.

Introduction

An alternative approach used in this paper for the analysis of flexural members is MIF. The method of initial function (MIF) is an analytical method of elasticity theory. The method makes it possible to obtain exact solutions of different types of problems, i.e., solutions without the use of hypotheses about the character of stress and strain.

According to this method, the basic desired functions are the displacements and stresses, the system of differential equations which are obtained from equations of Hook's law and equilibrium equations by replacing stresses by the displacements according to elasticity relations. The order of the derived equations depends on the stage at which the series representing the stresses and displacements are truncated. However, the physical significance of each term in the series is difficult to visualize, especially for the higher order terms.

Method of Initial Functions is used for the analysis of beams under symmetric central loading and uniform loading [1].The method of initial functions is used for the analysis of free vibration of rectangular beams of arbitrary depth. The frequency values are calculated using the Timoshenko beam theory and present the analysis for different values of Poisson's ratio[2]. The method of initial function is applied, to the flexural theory of circular plate subjected to antisymmetric lateral loads. The results are compared with solutions from classical theory[3]. They have used three-dimensional elasticity solutions for some static and dynamic problems of bending multi-layered anisotropic rectangular plates. They are derived by the method of initial functions[4]. Governing equations are developed for composite laminated deep beams by using method of initial functions. The beam theory developed can be used for beam sections of large depth[5]. Applied method of initial functions for the analysis of orthotropic deep beams and compared the results with the available theory[6]. Developed Hyperbolic Shear Deformation Theory for transverse shear deformation effects. It is used for the static flexure analysis of thick isotropic beams[7]. Used method of initial functions for the study of composite beams having two layers of orthotropic material and developed governing equation[8]. Method of initial functions is used for the analysis of composite laminated beams[9].

Formulation of the Problem

The equations of equilibrium for solids ignoring the body forces for two-dimensional case are:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \tag{1}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \tag{2}$$

The stress-strain relations for isotropic material are:

$$\sigma_x = C'_{11}\varepsilon_x + C'_{12}\varepsilon_y \tag{3}$$

$$\sigma_y = C'_{12}\varepsilon_x + C'_{22}\varepsilon_y \tag{4}$$

$$\tau_{xy} = C'_{33}\gamma_{xy} \tag{5}$$

The values of the coefficients C'_{11} to C'_{33} for isotropic materials are:

$$C'_{11} = C'_{22} = \frac{E}{1-\mu^2} \tag{6}$$

$$C'_{12} = \frac{\mu E}{1-\mu^2} \tag{7}$$

$$C'_{33} = G \tag{8}$$

The strain displacement relations for small displacements are:

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad (9)$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \qquad (10)$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \qquad (11)$$

Eliminating σ_x between equations (1) and (2) the following equations are obtained, which can be written in matrix form as

$$\frac{\partial}{\partial y}\begin{bmatrix} u \\ v \\ Y \\ X \end{bmatrix} = \begin{bmatrix} 0 & -\alpha & 0 & 1/G \\ C_1\alpha & 0 & C_2 & 0 \\ 0 & 0 & 0 & -\alpha \\ C_3\alpha^2 G & 0 & C_1\alpha & 0 \end{bmatrix}\begin{bmatrix} u \\ v \\ Y \\ X \end{bmatrix} \qquad (12)$$

Where,

$X = \tau_{xy}$, $Y = \sigma_y = C'_{12}\varepsilon_x + C'_{22}\varepsilon_y$

$C_1 = \frac{-a_{12}}{a_{22}}; C_2 = \frac{1}{Ga_{22}}; C_3 = \frac{a_{12}}{a_{22}} - a_{11}$ and

$a_{11} = \frac{C'_{11}}{G}, a_{12} = \frac{C'_{12}}{G}, a_{22} = \frac{C'_{22}}{G}$

The equation (12) can be expressed as:

$$\frac{\partial}{\partial y}\{S\} = [D]\{S\} \qquad (13)$$

The solution of equation (13) is

$$\{S\} = \left[e^{[D]y}\right]\{S_0\} \qquad (14)$$

Where $\{S_0\}$ is the vector of initial functions, being the value of the state vector $\{S\}$ on the initial plane.

If u_o, v_0 , Y_0 and X_0 are values of u ,v ,Y and X respectively, on the initial plane, then

$$\{S_0\} = [u_0, \quad v_0, \quad Y_0, \quad X_0]^T$$

(15)

Where

$$[L] = e^{[D]y} \qquad (16)$$

Expending (16) in the form of a series

$$[L] = [I] + y[D] + \frac{y^2}{2!}[D]^2 + ... \qquad (17)$$

Application of MIF

An isotropic beam of length l, depth, H and loaded with sinusoidal normal load $p = p_0\sin(\pi x/l)$ in the y- direction. The bottom plane of the beam is taken as the initial plane (p_0). Due to loading at the top plane of the beam one has

$X_0 = Y_0 = 0$

On the plane, y = H, the conditions are

X = 0, Y = -p

Y= -p on y=H, after simplification yields the governing partial differential equation:

$$(L_{Yu}.L_{Xv} - L_{Yv}.L_{Xu})\phi = -p \qquad (18)$$

Initial functions are obtained by substituting the value of φ:

$$u_0 = L_{Xv}\phi,$$
$$v_0 = -L_{Xu}\phi \qquad (19)$$

From the value of initial functions the value of displacements and stresses are obtained.

$$u = L_{uu}.u_0 + L_{uv}.v_0$$
$$v = L_{vu}.u_0 + L_{vv}.v_0$$
$$Y = L_{Yu}.u_0 + L_{Yv}.v_0$$
$$X = L_{Xu}.u_0 + L_{Xv}.v_0 \qquad (20)$$

Analysis of Flexural Member

The following values of beam dimensions are chosen for the particular problem,

H =100 cm, l = 400 cm

The following material properties are taken:

E=2.10x10^5 N/mm^2 μ = 0.30, G = 0.10x10^5 N/mm^2

The boundary conditions of the simply supported edges are:

$X = Y = v = 0$, at $x = 0$ and $x = l$

The boundary conditions are exactly satisfied by the auxiliary function.

$\Phi = A_1\sin(\pi x/l)$

A sinusoidal normal loading is assumed, on the top surface of the beam:

$P(x) = -P_0\sin(\pi x/l)$

Taking P_0 = 100N/mm and $x = l/2$

Initial functions are obtained from expression (19)

The values of u_0 and v_0 are substituted in expression (20) for obtaining the values of stresses and displacements.

u = 0.1274 cm

v = -0.2791cm

Y = -100.00N/mm^2

X = 0

Conclusion

The deflection obtained is also equal to the deflection obtained by other theories. The normal stress equal to the intensity of loading and shear stress equal to zero at the top of beam are obtain, this shows that MIF is successfully applied for the analysis of beam.

The beam theories based on MIF have advantage over other theories because of their capabilities to converge to an exact linear elasticity solution and so provide a governing equation of desired order according to the requirements of a beam problem. MIF gives accurate results in case of small thickness, large thickness and layered members. The MIF assumes a significant importance in the analysis of thick, composite or sandwich beams.

Notation

l	- Span of beam
H	- Total thickness of beam
E	- Young's modulus of Elasticity
G	- Shear modulus of Elasticity
μ	- Poisson's ratio
ε	- Strain
σ_x	- Bending stress
σ_y	- Normal stress
τ_{xy}	- Shear stress
u	- Displacements in x directions
v	- Displacements in y directions
α	- $\frac{\partial}{\partial x}$

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