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Mixed convection flow and heat transfer in a lid driven cavity using SIMPLE algorithm

Arti Kaushik
Department of Mathematics, Maharaja Agrasen Institute of Technology, Delhi, India
arti.kaushik@gmail.com

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Abstract

In the present study mixed convection flow and heat transfer in steady 2-D incompressible flow through a lid driven cavity is investigated. The upper wall of cavity is moving with uniform velocity and is at higher temperature. The stationary lower wall is kept at lower temperature. The governing equations of the model are solved numerically using SIMPLE algorithm. A staggered grid system is employed for numerical computations for velocity, pressure and temperature. Under relaxation factors for velocity, pressure and temperature are used for the stability of the numerical solutions. Bernoulli equation has been taken up to check the accuracy of the computed solutions. The significant findings from this study have been given under conclusion.

Keywords: Incompressible flow, lid driven cavity, SIMPLE algorithm, staggered grid system, mixed convection flow.

Introduction

The phenomena of combined forced convection and natural convection in a lid driven cavity have many practical applications[1-3]. The importance of this problem has led to various researches[4-9].

A numerical study of mixed convection heat transfer in a two-dimensional rectangular cavity with partially heated bottom wall and vertically moving sidewalls was done by Guo and Sharif[10]. Later, mixed convective heat transfer was studied in a lid driven cavity with aspect ratio 10, when the top of the lid is moving and is at higher temperature than the bottom wall[11]. Numerical simulations were done for 2-D lid-driven square enclosure partitioned by a solid divider with a finite thickness and finite conductivity to study the mixed convection heat transfer taking two different orientations of left vertical wall of enclosure[12]. Saha et al numerically studied two dimensional mixed convection in a square enclosure with moving top lid and keeping both top and bottom wall at constant temperature[13]. They studied flow and heat transfer characteristics, streamlines, isotherms and average wall Nusselt number for different Richardson number.

Using the finite volume method of the ANSYS FLUENT commercial CFD code laminar mixed convection characteristics were studied in a square cavity with a variable sized isothermally heated square blockage inside the cavity[14]. By keeping the blockage at a higher temperature and four surfaces of the cavity (including the lid) at a colder temperature, it was found that the blockage placed around the top left and the bottom right corners of the cavity results in the most preferred heat transfer. A study of steady laminar mixed convection inside a lid-driven square cavity filled with water, when both top and bottom walls are moving and are kept at cold and hot temperature respectively was done by Ismael et al[15]. They applied USR finite difference method and showed that convection is declined for certain critical values for the partial slip parameter. Using ANSYS FLUENT commercial code based on a finite volume method numerical study was done for mixed convection laminar flow in a lid-driven square cavity in which top lid of the cavity is moving rightwards[16]. The cavity had two square isothermally heated internal blockages which were kept at hot temperature and the walls of the cavity are kept at a cold temperature. They observed that the location of the blockage as well as the separation distance between the two blockages significantly changes the average Nusselt number. The effect of Richardson number on the heat transfer in a differentially heated lid–driven square cavity when top and bottom moving walls are maintained at different constant temperatures was studied[17]. Finite element approach using characteristic based split (CBS) algorithm was applied for this study. A numerically study of two-dimensional laminar mixed convection in a lid-driven square cavity filled with a nanofluid was done by Zeghbid and Bessaih[18]. They studied the effect of the Rayleigh number, the Reynolds number and the volume fraction of the nonofluid on the average Nusselt number using finite volume method. The movable top and bottom walls were kept at a local cold temperature and the nanofluid is constantly heated by two heat sources placed on the two vertical walls. Under these conditions it was found that the increase in Rayleigh number and solid volume fraction of nanofluids results in increase in average Nusselt number. Using ANSYS FLUENT, numerical investigation of two-dimensional conjugate heat transfer in a stepped lid-driven cavity under forced and mixed convection was done by Janjanam et al[19].

They studied three different nanoparticle volume concentrations of pure water and Aluminium oxide /water nanofluid and observed that mixed convection is 24% higher than that of forced convection for lower values of Reynolds number and higher values of Grashof number. The mixed convection flow in a tall lid driven cavity for non –Newtonian power law fluid was studied by Kumar et al[20]. They observed the effect of triangular surface corrugations on the flow and presented numerical results for different values of aspect ratio of cavity, Richardson numbers, Prandtl numbers (Pr), power-law indexes at a constant Grashof number. A laminar mixed convection in a square cavity with moving vertical wall and having a hot obstacle was studied by Ouahouah et al[21]. They investigated the effect of the Richardson number and Reynolds number on both hydrodynamic and thermal characteristics and observed that high values of Richardson and Reynolds numbers results in the enhanced heat transfer.

For the study of flow in lid driven cavity a well-known technique is SIMPLE algorithm proposed by Patankar and Spalding which has been used extensively in literature[22]. Sivakumar et al studied mixed convection heat transfer and fluid flow in lid-driven cavities with different lengths and locations of the heating portion using SIMPLE algorithm[23]. Using SIMPLE algorithm, an adaptive mesh refinement method was presented by Li and Wood for 2-D steady incompressible lid-driven cavity flows[24]. Their method is applicable to mathematical model containing continuity equations for incompressible fluid, steady state fluid flows or mass and heat transfer. A laminar two-dimensional lid driven cavity flow was investigated by Mohapatra with inclined side wall for different inclination angle using SIMPLE algorithm on staggered grid[25]. His results were in good agreement with the benchmark solutions. Comparison of SIMPLE and SIMPLER algorithm to study flow in a square cavity to analyze velocity and pressure distribution was done by Earn et al[26]. They showed that convergence rate of a numerical scheme is significantly affected by under-relaxation factors for velocity components and pressure. They also deduced that reduction of the heating portion results in increased heat transfer rate. Ali et al studied the heat transfer rate in a double lid-driven rectangular cavity keeping bottom wall is kept at a high temperature and the top wall is kept at a low temperature using SIMPLE algorithm[27]. They employed hybrid nanofluid and showed that the presence of hybrid nanofluid in the rectangular cavity increases the heat transfer significantly.

In the view of above mentioned literature, it is clear that SIMPLE algorithm has not been applied much to study mixed convective heat transfer in a lid driven cavity with moving upper wall which is at higher temperature than the bottom wall. Our aim in this paper is to examine the mixed convection and heat transfer in a lid-driven cavity under these conditions using SIMPLE algorithm. The results obtained are shown through quiver plot and contour plots for various considered dimensionless parameters.

Methodology

We consider a two dimensional, incompressible, steady and laminar flow through a square cavity of height and length L. The walls of the cavity have no-slip condition except the upper wall which is moving in its own plane at a constant speed U_0. The cavity upper wall is kept at a high temperature T_h whereas bottom wall is kept at a low temperature T_c, The left and right walls are assumed to be adiabatic. All thermo-physical properties of the fluid are considered as constant except the density variation of the buoyancy term. The density is assumed to vary linearly with temperature as $\left(\rho - \rho_c\right) = g\beta\left(T - T_c\right)$, where ρ is the fluid density, g is the acceleration due to gravity and β is the coefficient of thermal expansion. The fluid is considered as Newtonian. We also assume that viscous dissipation is neglected.

Under these assumptions, the governing conservations equations are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta\left(T - T_c\right) \qquad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (4)$$

where u and v are the fluid velocity components in the x- and y directions, p the pressure, T the temperature, g is acceleration due to gravity, β the volumetric coefficient of thermal expansion, υ is kinematic viscosity, ρ the density of the fluid and α the thermal diffusivity.

Introducing the non-dimensional quantities

$$X = \frac{x}{L},\ Y = \frac{y}{L},\ U = \frac{u}{U_0},\ V = \frac{v}{U_0},\ \theta = \frac{T - T_c}{T_h - T_c},\ \mathrm{P} = \frac{p}{\rho U_0^{\,2}} \qquad (5)$$

we can write Eqn. (1)-(4) in non-dimensional form as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad (6)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \qquad (7)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\text{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{Gr}{\text{Re}^2}\theta \qquad (8)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\text{Re}\,\text{Pr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (9)$$

Where Re, Pr and Gr are non- dimensionalised Reynolds number, Prandtl number and Grash of number respectively and are defined as

$$\text{Re}=\frac{U_0 L}{\upsilon}3 \;,\; \text{Pr}=\frac{\upsilon}{\alpha} \;,\; Gr=\frac{g\beta\Delta T L^3}{\upsilon^2}$$

The initial and boundary conditions in the non dimensionalised form are

$U=1\;, V=0,\; \theta=1 \qquad 0\le X\le 1\,, Y=1$

$U=0\;, V=0,\; \theta=0 \qquad 0\le X\le 1\,, Y=0$

$U=V=0\;,\; \frac{\partial\theta}{\partial X}=0 \qquad 0\le Y\le 1\,, X=0\;\&\; X=1$

The governing equations along with the boundary conditions are solved using SIMPLE algorithm. The algorithm was originally put forward by Patankar and Spalding. It is a guess and correct procedure for the calculation of pressure and velocities. For the application of the technique the computational domain is discredited employing the staggered grid arrangement. Then using finite volume method the nonlinear governing partial differential equations are converted into a system of discredited equations. These discredited equations along with pressure correction equation are solved to obtain the velocities, pressure and temperature at all node points of the grid.

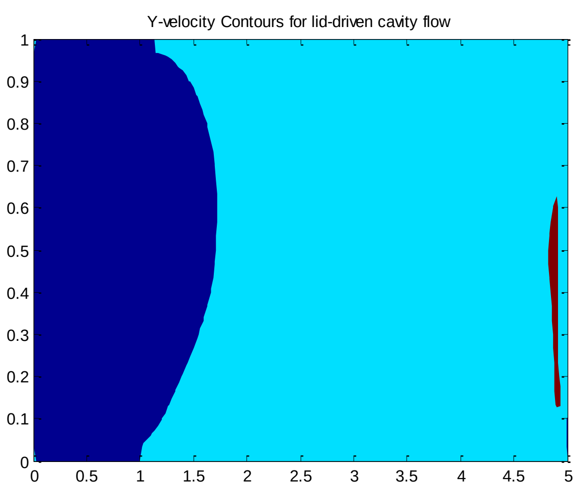
Results and Discussion

The governing equations are solved numerically to obtain unknown variables u, v, p and T for various values of α, β, ρ and μ. The SIMPLE algorithm has been implemented in MATLAB programming language. During the numerical process, different values of under relaxation factors for u velocity and v velocity and pressure were tested for all cases. The effect of thermal diffusivity α on v velocity is shown in Figure-1. It can be seen that increase in value of α results in higher v- velocity. For lower values of α, the v velocity has sudden increase near midpoints of east boundary.

In Figure-2, the variation in temperature with change in α can be seen. For lower values of α, the temperature start decreasing from north boundary and then remains same in the cavity. For higher values of α, this pattern remains same but then a sudden increase in temperature can be seen near south east corner of the cavity.

The effect of coefficient of thermal expansion β on the u velocity and v velocity are shown in Figure-3 and 4. It can be seen that decrease in value of β results in higher u - velocity and v- velocity. The values of u velocity increase slowly from north boundary till midpoints of the cavity and then decrease rapidly towards the south boundary. This pattern remains same for all values of β. The v-velocities are highest near midpoints along east boundary and lowest near west boundary.

In Figure-5, the pressure contours for different values of β can be seen. The pressure is high for all point in cavity but decreases near the south boundary of the cavity. Note that for lower value of β the pressure in the cavity is higher. For higher value of β the pressure decreases slightly near east and west boundary. The change in temperature inside the cavity with change in β is minimal. This can be seen in Figure-6.



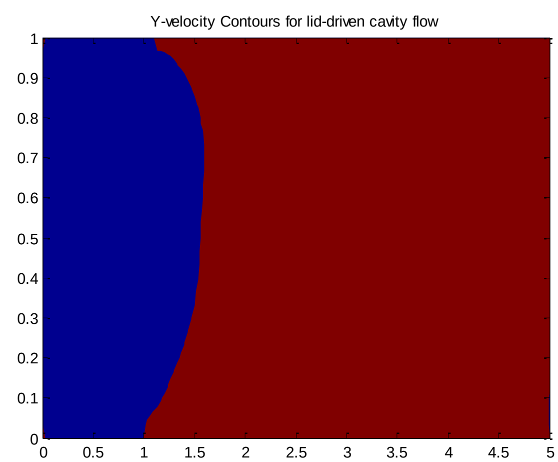


Figure-1: Contour plot for v velocity for α=10 and α=50.

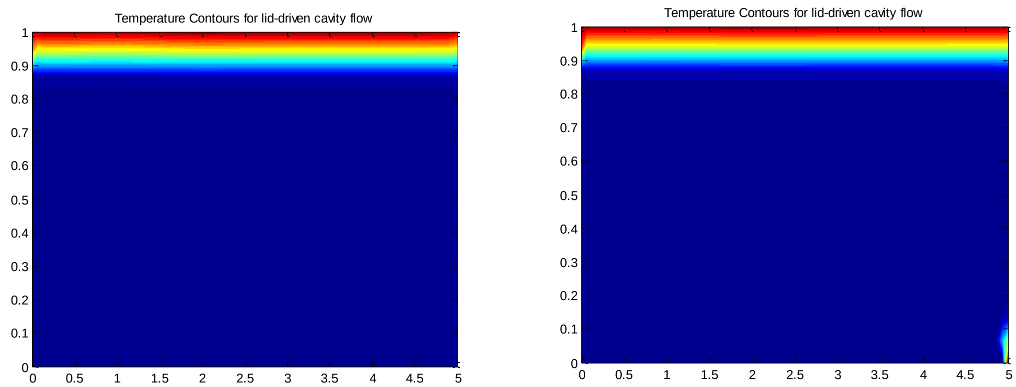


Figure-2: Temperature contours for α=10 and α=50.

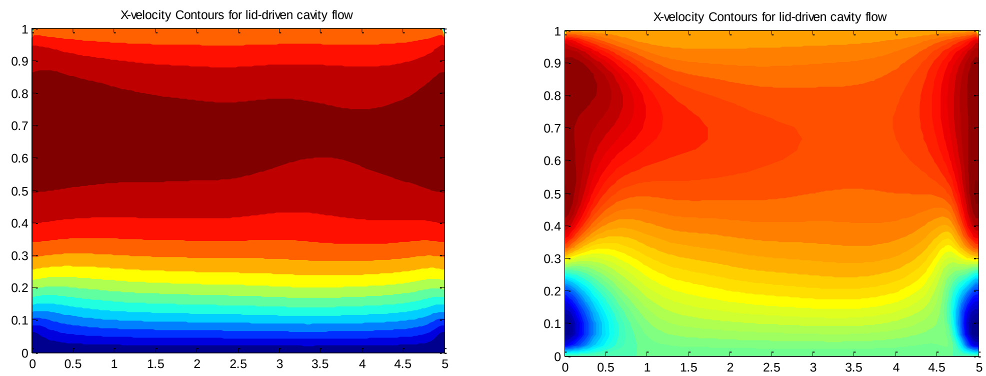


Figure-3: Contour plot for u velocity for β=0.00001 and β=0.0001.

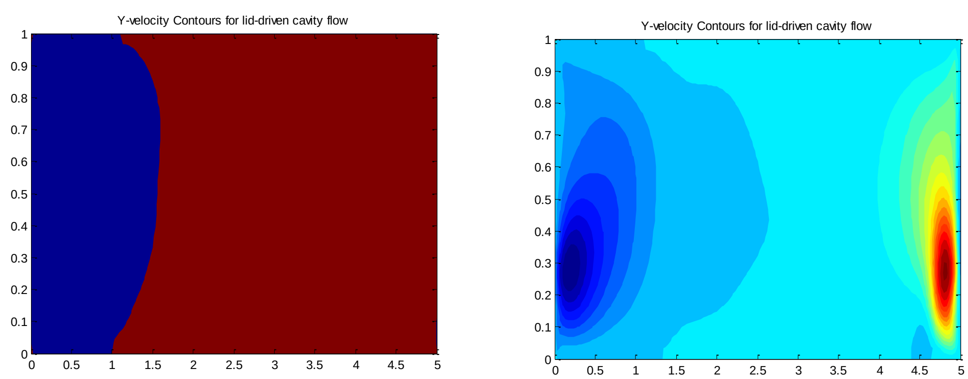


Figure-4: Contour plot for v velocity for β=0.00001 and β=0.0001.

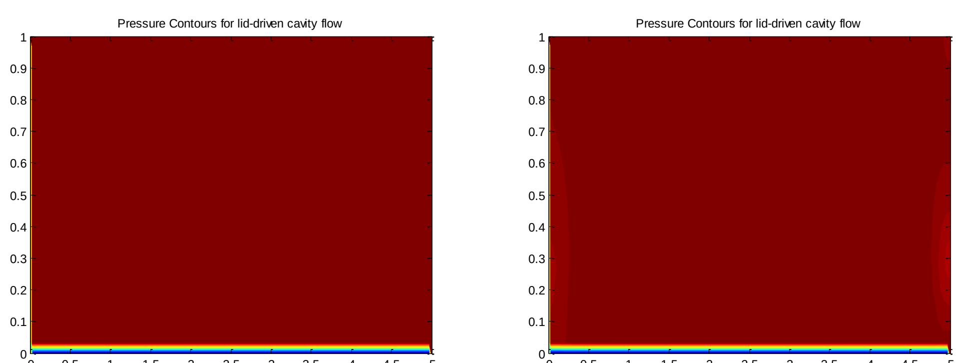


Figure-5: Pressure Contour plot for β=0.00001 and β=0.0001.

Figure-7 and 8 shows the effect of density of fluid ρ on the u velocity and v velocity, respectively.

It can be seen that increase in value of ρ results in higher u -velocity and v- velocity. For higher values of ρ, u velocity increase slowly from north boundary till midpoints of the cavity and then decrease rapidly towards the south boundary. For lower values of ρ, u velocity increase from north boundary till midpoints of the cavity and then decrease towards the south boundary in a symmetric manner. The v-velocities are higher near midpoints along east and west boundaries and lowest near south east corner. In Figure-9, we can see as expected that the pressure in the cavity decreases with decreasing density. Figure-10 shows that the temperature at south east corner increases in magnitude as the density ρ increases. Figure-11 shows the effect of density of fluid on the direction field of the flow. As the density of the fluid increases, the direction field of the flow changes drastically.

The effect of viscosity of the fluid on u velocity and v velocity can be seen in Figure-12 and 13, respectively. It can be seen that the pattern of increase of u velocity from north boundary and decrease towards south boundary remains same, but varies as the viscosity changes. For higher viscosity the maximum u-velocity lies near midpoints of east and west boundaries. For v velocity, we can see that it has minimum values near west boundaries. For higher viscosity, the value of v velocity also has lower values near north boundaries.

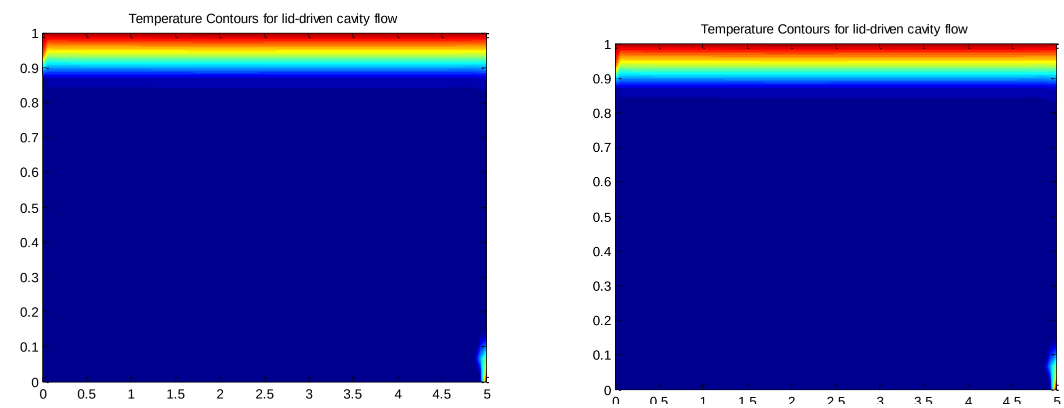


Figure-6: Temperature contours for β=0.0001 and β=0.00001.

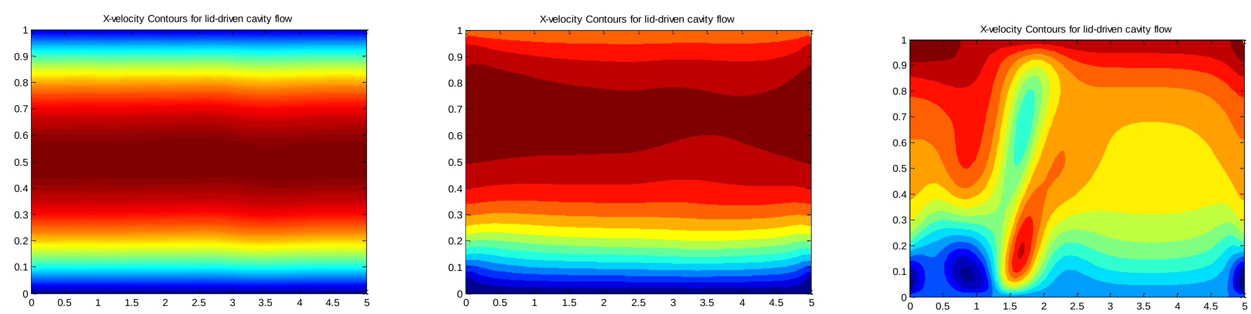


Figure-7: Contour plot u velocity for ρ=1, ρ=10 and ρ=50.

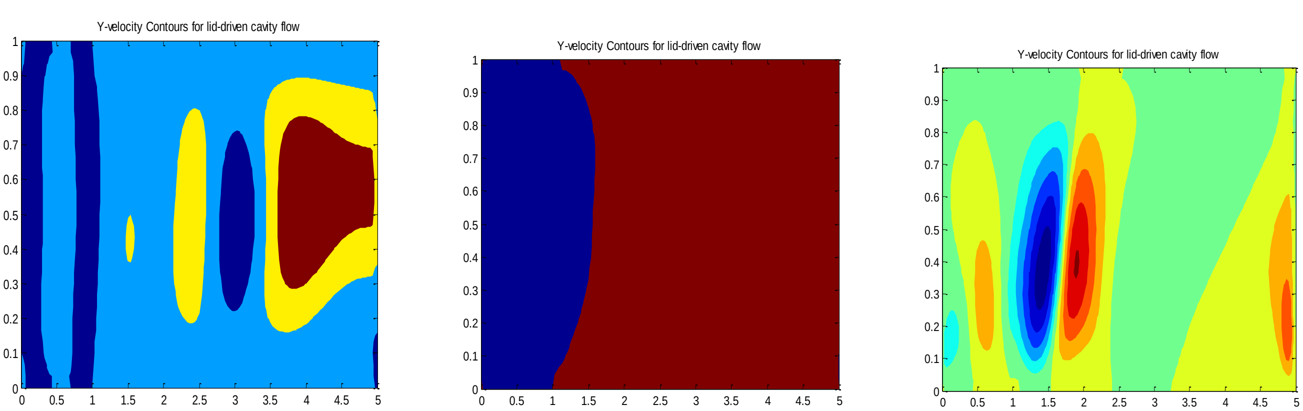


Figure-8: Contour plot v velocity for ρ=1, ρ=10 and ρ=50.

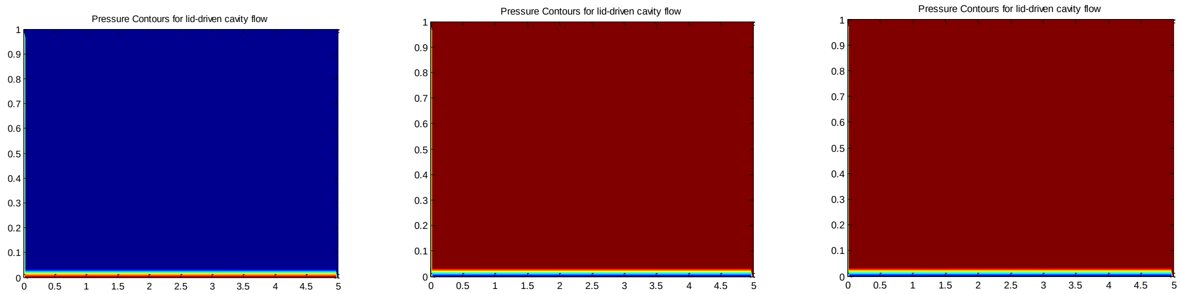


Figure-9: Pressure Contour for ρ=1, ρ=10 and ρ=50.

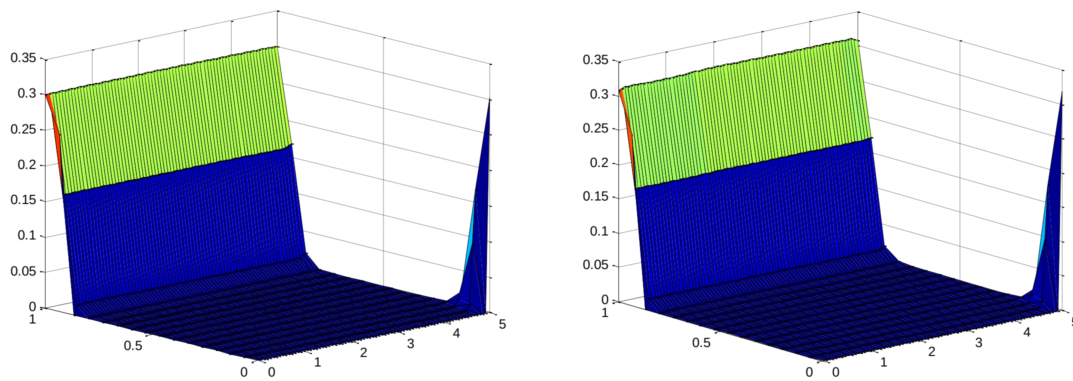


Figure-10: Temperature variation at all node points for ρ=1 and ρ=10.

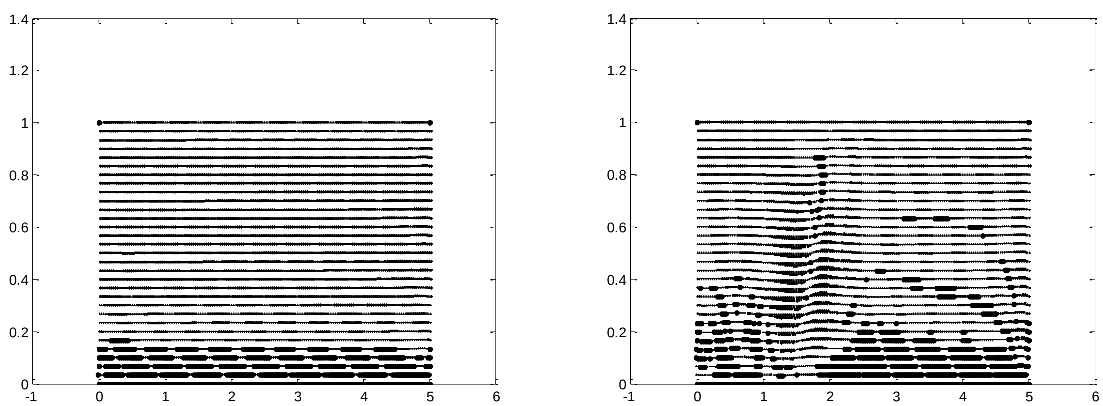


Figure-11: Direction field of the flow for ρ=10 and ρ=50.

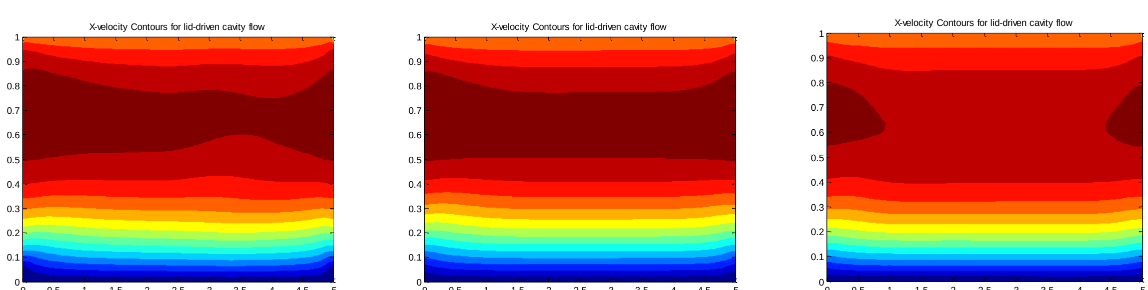


Figure-12: Contour plots for u velocity for μ=0.01, μ=0.05 and μ=0.1.

The variation of pressure inside the cavity with varying viscosity can be seen in Figure-14. It can be clearly seen that the pressure increases with increasing viscosity of the fluid. The change in temperature with change in viscosity is minimal.

The effect of velocity of the top boundary on u- velocity can be seen in Figure-15. As the velocity of top boundary increases, u- velocity decreases. For higher velocity of top boundary, the maximum values of u- velocity shifts to north boundary of the cavity from centre of the cavity. On the other hand, the v velocity first decreases and then starts increasing again, with increasing velocity of top boundary. This is shown in Figure-16.

In Figure-17, we can see that the pressure in the cavity decreases with increase in the velocity of top boundary. In Figure-18, we can see the change in direction field of the flow with increasing velocity of top boundary.

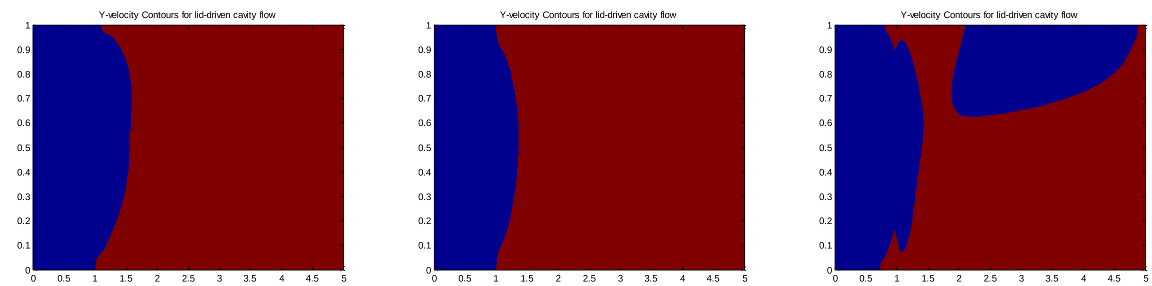


Figure-13: Contour plots for v velocity for μ=0.01, μ=0.05 and μ=0.1.

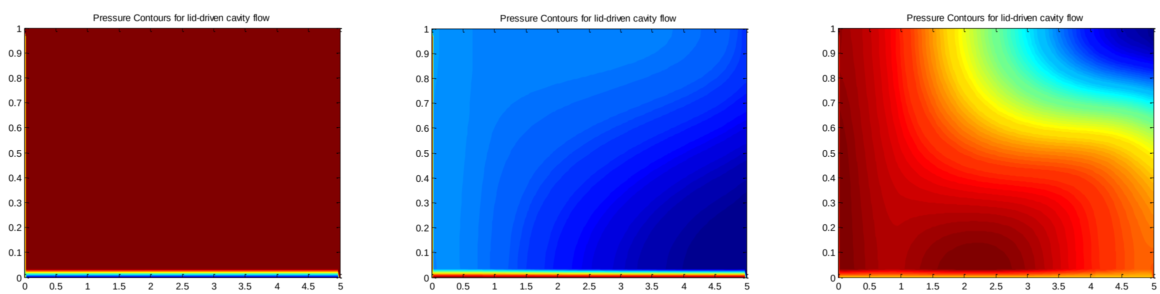


Figure-14: Pressure Contour plots for μ=0.01, μ=0.05 and μ=0.1.

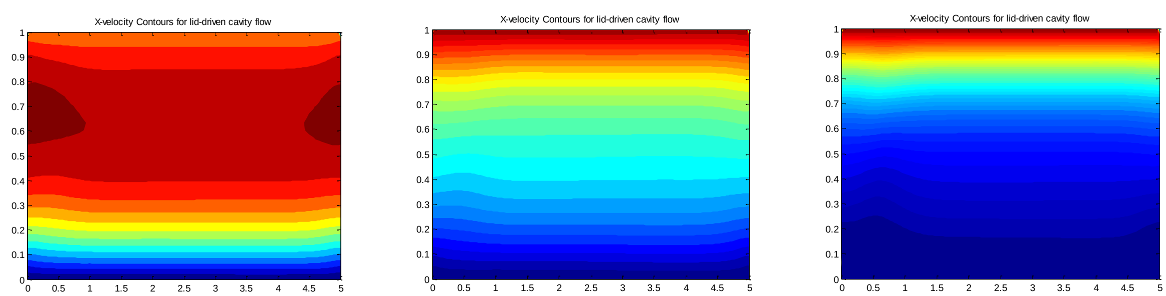


Figure-15: Contour for u velocity for wall velocity 0.1, 0.25 and 0.5.

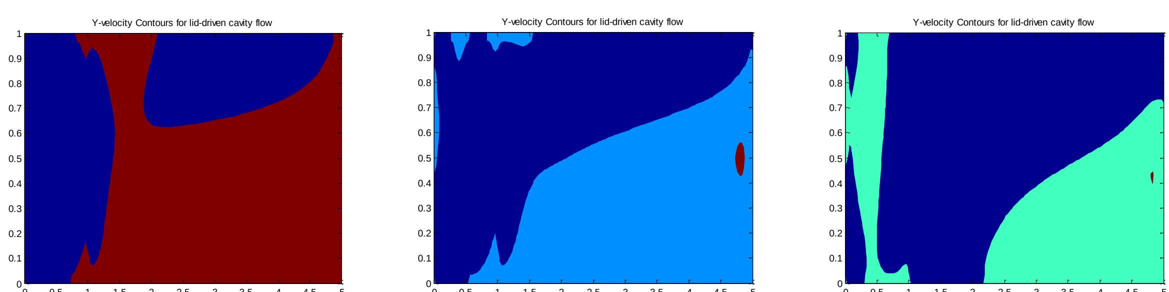


Figure-16: Contour for v velocity for wall velocity 0.1, 0.25 and 0.5.

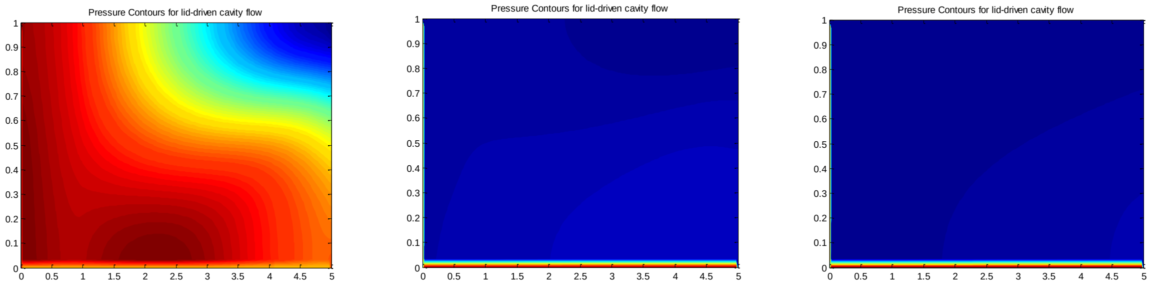


Figure-17: Pressure contour for wall velocity 0.1, 0.25 and 0.5.

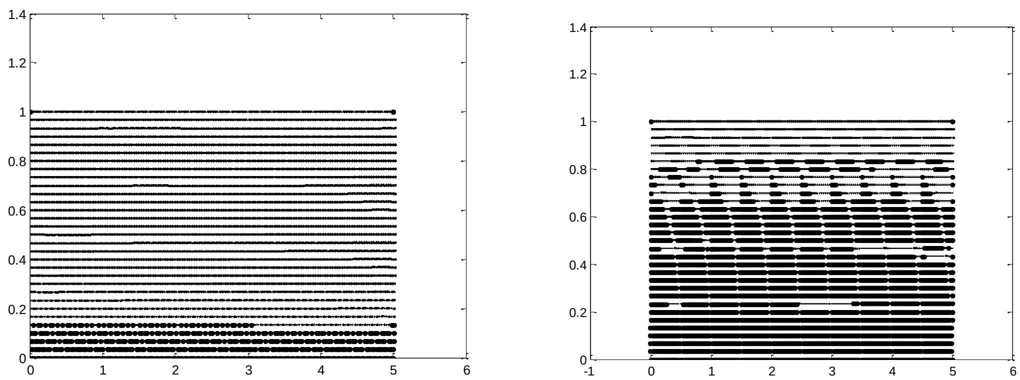


Figure-18: Direction fields for wall velocity 0.1and 0.5.

Conclusion

This paper presents a numerical study of mixed convection flow and heat transfer in steady 2-D incompressible flow through a lid driven cavity. Numerical solutions for the governing equations are obtained using SIMPLE algorithm. The numerical computations for u-velocity, v velocity and pressure were conducted using staggered grid system. It was seen that the solution converges for few values of under relaxation factor for pressure. It was seen that the u-velocity and v- velocity increase with increase in density of the fluid, but decreases with increase in the value of coefficient of thermal expansion. Also the location of maximum values of u –velocity and v velocity changes drastically when the viscosity of the fluid changes. The direction fields of the flow were also discussed for various cases. It was found that the direction field of the flow changes significantly when the density of the fluid changes or the velocity of top wall increases. It was observed that the pressure inside the cavity increases with increase in density or viscosity of the fluid, whereas it decreases with increase in the value of coefficient of thermal expansion. It was also observed that the temperature varies with change in thermal conductivity and density of fluid, but no change can be seen with change in coefficient of thermal expansion and viscosity of the fluid.

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