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Atherosclerotic Study of non-Isothermal non-Newtonian Steady Flow of Blood in a Plane by Adomian Decomposition Method

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Abstract

In the present study, the analytical solutions of blood flow for two dimensional non-isothermal, non-Newtonian fluids flowing through the channel having symmetric stenosis of cosine shape are discussed. The governing Navier-Stokes equations are reduced to compatibility equation along with energy equation and solved analytically by Adomian decomposition method (ADM) and regular perturbation method (RPM). The results are presented analytically and graphically in terms of streamlines, wall shear stress, separation and reattachment points and temperature distribution on blood flow through a stenoised channel. It has been observed that the non-Newtonian nature of blood reduces the magnitude of the peak of flow over the stenoised region. Further, increase in second grade parameter (α) *increases the temperature and wall shear stress while the critical Re decreases. It is observed from comparison that the ADM is efficient, reliable, easily computable and provides a fast convergent series. It worthy noting that the results obtained in this paper are compared with published results and found good agreement.*

Keywords: Second grade fluid, ADM, RPM, Heat transfer, Wall shear stress.

Introduction

The present analysis is concerned with the solution of non-linear, two dimensional, compatibility and energy equations by Adomian decomposition method and regular perturbation method. The powerful tool for calculating the analytic series solution of linear and nonlinear partial differential equations is ADM, which was introduced and developed by George Adomian and is well addressed in the literature[1-4]. Considerable research work is done recently by applying ADM to linear and nonlinear equations. Bellomo and Monaco gives a useful comparison between the ADM and perturbation method proved the efficiency of the decomposition method compared over perturbation method[5].

It is well known that the stenosis is a disease of arteries and is caused due to abnormal growth in the lumen of the artery. Its actual cause may not be known exactly but its effect on cardiovascular system can easily be understood by analyzing the blood flow in its vicinity. One of the practical applications of blood flow through a membrane oxygenator is the flow with an irregular wall surface. Many authors studied the behavior of blood in a constricted artery by considering different models of stenosis and assuming the blood to be Newtonian and non-Newtonian fluid.

One of the earliest studies in this regard was conducted by Young[6]. He considered the blood as Newtonian fluid and suggested that the irregular walls can be an important factor in the development of arterial diseases. Forrester and Young presented the analytical solution of Newtonian fluid for an axisymmetric, steady, incompressible flow and considered mild constriction for the flow of blood, both theoretically and experimentally in the converging and diverging tube[7]. Lee and Fung solved the flow model of the Newtonian fluid numerically through locally constricted tube for the low Reynolds number[8]. Morgan and Young carried out the extension of Young[9]. They used an integral method and presented the approximate analytical solution of axisymmetric, steady state flow, which is applicable to both a mild and severe constriction. Haldar investigated the flow of blood through an axisymmetric constricted artery of cosine shape and presented the solutions for velocity, wall shear stress and separation point[10]. He indicated the presence of separation point, due to the occurrence of negative wall shear stresses at high Reynolds numbers. Chow et al. analyzed the laminar flow of incompressible steady Newtonian fluid for different physical parameters by considering the sinusoidal boundary[11]. It is observed that by increasing either *Re* or ε, the separation point would move down towards the throat in the divergent part of the channel with subsequent enlargement of the region of separation.

In addition to the Newtonian model many authors have studied the behavior of blood as non-Newtonian fluid. The non-Newtonian fluid may be considered as comparatively better model to represent the blood, due to its cells suspension property, even at a low shear rate. Further the Newtonian model is reasonable with regards the large channel assumption.

However, for smaller channels the flow is expected to take on non-Newtonian character. Shukla et al. presented the analysis of blood by considering it as non-Newtonian fluid and studied the effect of constriction on the resistance to flow along with wall shear stress in an artery[12]. Vahdati et al. designed a non linear ordinary differential equation for non-fatal disease in population and solved by Homotopy analysis method[13]. Thundil and Ramsai assumed the fluid to be air and presented numerical investigation using CFD[14]. Chauhan et al. studied the effect of turbulent flow over Ahmed's body by applying numerical technique[15].

It should be noted that all the above investigations are limited to flow patterns, separation and reattachment points. However, the present work also investigates the effect of heat transfer in the channel. The solutions are presented graphically in terms of stream lines, wall shear stress, points of separation and reattachment and temperature distribution. It is assumed that the blood flow is time independent between two parallel plates, situated at the separation $2h_0$. We observe that the governing equations are highly non-linear and apply the ADM and perturbation technique having δ as a small parameter to find the analytical solution.

Problem Formulation

It is assumed that the blood behaves like homogeneous, incompressible, non-Newtonian fluid of second grade with heat transfer. The governing equations for the present analysis are conservation of mass, momentum and energy equation. Consider the steady flow of blood through the channel of infinite length having stenosis of length $l_o/2$. The coordinate system is constituted in such a way that the channel lies in xy-plane and x-axis coincide with the center line in the direction of flow and y-axis perpendicular to x-axis. Consider the boundary of the stenoised region of the form Haldar[5] as

$$h(\tilde{x}) = h_o - \frac{\lambda}{2}\left(1+\cos\left(\frac{4\pi\tilde{x}}{l_o}\right)\right), \qquad -\frac{l_o}{4} < \tilde{x} < \frac{l_o}{4}, \tag{1}$$

$$= h_o, \qquad \text{otherwise},$$

where $h(\tilde{x})$ is the variable width of channel, $2h_o$ the width of unobstructed channel and λ the maximum height of stenosis. Geometry of the problem is shown in Figure-1.

It is assumed that the blood behaves like non-Newtonian fluid and for steady, homogeneous, incompressible two dimensional flow of blood velocity field is taken as

$$\tilde{V} = (\ \tilde{u}(\tilde{x},\tilde{y}),\ \tilde{v}(\tilde{x},\tilde{y}),\ 0\ ) \tag{2}$$

Introducing the dimensionless quantities of the form

$$x = \frac{\tilde{x}}{l_o},\quad y = \frac{\tilde{y}}{h_o},\quad u = \frac{\tilde{u}}{u_o},\quad v = \frac{\tilde{v}}{u_o},\quad p = \frac{h_o^2}{\mu u_o l_o}\tilde{p},\quad \theta = \frac{\tilde{T}-T_o}{T_1 - T_o} \tag{3}$$

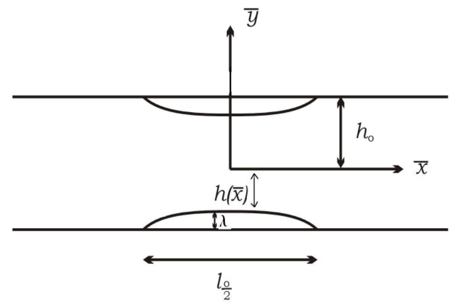

Figure-1
Geometry of the problem

Where u_o is the characteristic velocity and T_1, T_o are temperature on the boundary of stenosis and fluid respectively. Dimensionless form of the boundary profile becomes

$$f = 1 - \frac{\varepsilon}{2}\left(1+\cos 4\pi x\right), \qquad -\frac{1}{4} < x < \frac{1}{4} \qquad = 1, \tag{4}$$

otherwise,

Where: $f = h(\tilde{x})/h_o$ and $\varepsilon = \lambda / h_o$ is dimensionless height of stenosis. Introducing the stream functions of the form

$$u = \frac{\partial\psi}{\partial y},\quad v = -\delta\frac{\partial\psi}{\partial x} \tag{5}$$

which satisfy the continuity equation identically and compatibility equation along with energy equation in terms of stream function takes the form

$$\mathrm{Re}\,\delta\frac{\partial(\psi,\nabla^2\psi)}{\partial(y,x)} = \nabla^4\psi + \alpha\delta\frac{\partial(\psi,\nabla^4\psi)}{\partial(y,x)}, \tag{6}$$

$$Pe\delta\frac{\partial(\psi,\theta)}{\partial(y,x)} = \nabla^2\theta + Br\left(1+\frac{\alpha}{2}\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\right)$$

$$\left(4\delta^2\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2 + \left(\frac{\partial^2\psi}{\partial y^2} - \delta^2\frac{\partial^2\psi}{\partial x^2}\right)^2\right) \tag{7}$$

Boundary conditions in terms of stream functions for velocity component and temperature are

$$\frac{\partial\psi}{\partial y} = 0,\quad \psi = -\frac{1}{2},\quad \theta = 1 \quad at \quad y = f, \quad and$$

$$\frac{\partial^2\psi}{\partial y^2} = 0,\ \psi = 0,\ \frac{\partial\theta}{\partial y} = 0 \quad at \quad y = 0. \tag{8}$$

Where: $\nabla^2 = \delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and

$$\alpha=\frac{\alpha_1 u_o}{\mu h_o},\quad \delta=\frac{h_o}{l_o},\quad \mathrm{Re}=\frac{u_o h_o}{\nu},$$

$$\mathrm{Br}=\frac{u_o^2\mu}{k\left(T_1-T_o\right)},\quad \mathrm{Pe}=\frac{\rho u_o h_o c_p}{k}.$$

It is noted that for $\alpha=0$, the above model reduces to Newtonian case and reduced compatibility Equation-6 for $\alpha=0$ has been discussed by many authors.

Solution

The resulting compatibility and energy equations are non-linear and exact solution is very difficult to find, for the series solutions we apply ADM and RPM in these equations by considering δ as a small parameter for RPM as follows.

Solution of compatibility equation by ADM: The compatibility equation is

$$L\psi=\delta A_n-\delta^4\frac{\partial^4\psi}{\partial x^4}-2\,\delta^2\frac{\partial^4\psi}{\partial x^2\partial y^2} \tag{9}$$

Where: $L=\dfrac{\partial^4}{\partial y^4}$ (10)

along with the invertible expression is $L^{-1}=\int\int\int\int(*)dydydydy.$ (11)

Where $A_n=\mathrm{Re}\dfrac{\partial\left(\psi,\nabla^2\psi\right)}{\partial(y,x)}-\alpha\dfrac{\partial\left(\psi,\nabla^4\psi\right)}{\partial(y,x)}$ (12)

Apply L^{-1} to equation (9), we get

$$\psi=\frac{A}{6}y^3+\frac{B}{2}y^2+C\,y+D+L^{-1}\left[\delta A_n-\delta^4\frac{\partial^4\psi}{\partial x^4}-2\,\delta^2\frac{\partial^4\psi}{\partial x^2\partial y^2}\right] \tag{13}$$

Where: A, B, C, D are functions of x to be determined. For the series solution, substituting

$$\psi=\sum_{n=0}^{\infty}\psi_n(x,y),\quad A_n=\sum_{n=0}^{\infty}A_n \tag{14}$$

in Equation-13, we obtain

$$\sum_{n=0}^{\infty}\psi_n(x,y)=\frac{A}{6}y^3+\frac{B}{2}y^2+C\,y+D+L^{-1}\left[\delta\sum_{n=0}^{\infty}A_n-\delta^4\frac{\partial^4\left(\sum_{n=0}^{\infty}\psi_n(x,y)\right)}{\partial x^4}-2\,\delta^2\frac{\partial^4\left(\sum_{n=0}^{\infty}\psi_n(x,y)\right)}{\partial x^2\partial y^2}\right] \tag{15}$$

and boundary conditions becomes

$$\frac{\partial\left(\sum_{n=0}^{\infty}\psi_n(x,y)\right)}{\partial y}=0,\quad \sum_{n=0}^{\infty}\psi_n(x,y)=-\frac{1}{2}\quad at\quad y=f,$$

$$\frac{\partial^2\left(\sum_{n=0}^{\infty}\psi_n(x,y)\right)}{\partial y^2}=0,\quad \sum_{n=0}^{\infty}\psi_n(x,y)=0\quad at\quad y=0. \tag{16}$$

Comparing both sides of Equation-15 and 16, we obtain

$$\psi_o=\frac{A}{6}y^3+\frac{B}{2}y^2+C\,y+D \tag{17}$$

$$\frac{\partial\psi_o}{\partial y}=0,\quad \psi_o=-\frac{1}{2}\quad at\quad y=f,\quad and$$

$$\frac{\partial^2\psi_o}{\partial y^2}=0,\quad \psi_o=0\quad at\quad y=0. \tag{18}$$

The solution of (17) is obtained by using the boundary conditions on stream function from (18) as follows

$$\psi_o=\frac{\eta}{4}\left(\eta^2-3\right),\qquad \eta=\frac{y}{f} \tag{19}$$

it is observed that the expression of ψ_o for both the methods is same. Now the recursive relation from (15) becomes as

$$\psi_{n+1}=L^{-1}\left[\delta A_n-\delta^4\frac{\partial^4\psi_n}{\partial x^4}-2\,\delta^2\frac{\partial^4\psi_n}{\partial x^2\partial y^2}\right] \tag{20}$$

and the expression for ψ_1 and ψ_2 along with boundary conditions are given by

$$\psi_1=L^{-1}\left[\delta A_o-\delta^4\frac{\partial^4\psi_o}{\partial x^4}-2\,\delta^2\frac{\partial^4\psi_o}{\partial x^2\partial y^2}\right] \tag{21}$$

$$\frac{\partial\psi_1}{\partial y}=0,\quad \psi_1=0\quad at\quad y=f,\quad and$$

$$\frac{\partial^2\psi_1}{\partial y^2}=0,\quad \psi_1=0\quad at\quad y=0 \tag{22}$$

and $\psi_2=L^{-1}\left[\delta A_1-\delta^4\dfrac{\partial^4\psi_1}{\partial x^4}-2\,\delta^2\dfrac{\partial^4\psi_1}{\partial x^2\partial y^2}\right]$ (23)

$$\frac{\partial\psi_2}{\partial y}=0,\quad \psi_2=0\quad at\quad y=f,\quad and$$

$$\frac{\partial^2\psi_2}{\partial y^2}=0,\quad \psi_2=0\quad at\quad y=0 \tag{24}$$

Where: $A_o=\mathrm{Re}\dfrac{\partial\left(\psi_o,\nabla^2\psi_o\right)}{\partial(y,x)}-\alpha\dfrac{\partial\left(\psi_o,\nabla^4\psi_o\right)}{\partial(y,x)}$ (25)

$$A_1 = \mathrm{Re}\frac{\partial(\psi_o, \nabla^2\psi_1)}{\partial(y,x)} + \mathrm{Re}\frac{\partial(\psi_1, \nabla^2\psi_o)}{\partial(y,x)} -$$

and

$$\alpha\frac{\partial(\psi_o, \nabla^4\psi_1)}{\partial(y,x)} - \alpha\frac{\partial(\psi_1, \nabla^4\psi_o)}{\partial(y,x)}. \quad (26)$$

The solution of ψ_1 is obtained from (21) by substituting ψ_o along with boundary conditions from (22) as

$$\psi_1 = -\frac{\delta\eta(\eta^2-1)^2}{26880f^2}$$
$$[\ -96\alpha\delta^{2,3}\left(36(\eta^2-5)+\delta^2 f'^2(\ 25\eta^4-58\eta^2-15\ )\right)$$
$$+24\alpha\delta^2 ff' f''(\ 99(\eta^2-5)$$
$$+\delta^2 f'^2(\ 175\eta^4-433\eta^2-30\ )\ )$$
$$+8f^2\{\ \mathrm{Re}\delta^2 f'^3(\ 5\eta^4-17\eta^2+24\ )$$
$$+72\,\delta^3 f'^4(5\eta^2+3)+3f'(\ 3\mathrm{Re}(\eta^2-5)$$
$$-\alpha\delta^4 f''^2(\ 55\eta^4-151\eta^2+84\ )\ )-27\alpha\delta^2 f^{(3)}$$
$$+f'^2(\ 1008\delta-\alpha\delta^4 f^{(3)}(\ 95\eta^4-269\eta^2+186\ )\ )\ \}$$
$$-24\delta^3 f^5 f^{(4)}(\eta^2-5)+\delta f^3$$
$$\{\ f''(\ -9\mathrm{Re}\delta f'(\ 5\eta^4-18\eta^2+29\ )$$
$$-288\delta^2 f'^2(10\eta^2-1)+8(\ 5\alpha\delta^3 f^{(3)}(\ 5\eta^4$$
$$-17\eta^2+24\ )-252\ )\ )+\alpha\delta^3 f'^{(4)}(\ 85\eta^4-298\eta^2+453\ )\ \}$$
$$+\delta^2 f^4\{\ 144\delta f''^2(2\eta^2-3)$$
$$+f^{(3)}(\ \mathrm{Re}(\ 5\eta^4-26\eta^2+69\ )+192\delta f'(2\eta^2-3)\ )$$
$$-\alpha\delta^2\ f^5(\ 5\eta^4-26\eta^2+69\ )\ \}\ ]. \quad (27)$$

Similarly the solution for ψ_2 up to second order is obtained by substituting ψ_1 and ψ_o in equation (23) along with respective boundary conditions from (24), we get

$$\psi_2 = \frac{\mathrm{Re}\eta(\eta^2-1)^2}{3449600f^2}[\ f'^2\{\ -1540\alpha(\ 5\eta^4-26\eta^2+69\ )+$$
$$\mathrm{Re}f^2(\ 98\eta^6-959\eta^4+2492\eta^2-2875\ )\ \} \quad (28)$$
$$+ff''\{\ 385\alpha(\ 5\eta^4-26\eta^2+69\ )-\mathrm{Re}\ f^2(\ 35\eta^6-315\eta^4+$$
$$853\eta^2-1213\ )\ \}\ ]+\ .higher\,order\,of\ \delta.$$

It is found that the solutions for ψ_1 and ψ_2 by ADM are different from RPM. Now we can obtain the velocity components u, v from Equations-5 and 14 easily.

Solution of Energy Equation by ADM: Dimensionless form of energy equation in terms of stream function is

$$L_1\theta = Pe\delta\frac{\partial(\psi,\theta)}{\partial(y,x)} - Br\left(1+\frac{\alpha}{2}\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\right)$$
$$\left(4\delta^2\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2+\left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)^2\right)-\delta^2\frac{\partial^2\theta}{\partial x^2} \quad (29)$$

Where $L_1 = \dfrac{\partial^2}{\partial y^2}$ (30)

is highest order derivative and its inverse is defined as

$$L_1^{-1} = \iint(*)dydy. \quad (31)$$

Apply L_1^{-1} to equation (29), we get

$$\theta = C_1 y + C_2 + L_1^{-1}\left[\begin{array}{l} Pe\delta\dfrac{\partial(\psi,\theta)}{\partial(y,x)} - Br\left(\begin{array}{c}1+\dfrac{\alpha}{2}\delta \\ \left(\dfrac{\partial\psi}{\partial y}\dfrac{\partial}{\partial x}-\dfrac{\partial\psi}{\partial x}\dfrac{\partial}{\partial y}\right)\end{array}\right) \\ \left(4\delta^2\left(\dfrac{\partial^2\psi}{\partial x\partial y}\right)^2+\left(\dfrac{\partial^2\psi}{\partial y^2}-\delta^2\dfrac{\partial^2\psi}{\partial x^2}\right)^2\right)-\delta^2\dfrac{\partial^2\theta}{\partial x^2}\end{array}\right] \quad (32)$$

where C_1 and C_2 are functions of x to be determined.

Substituting $\psi = \sum_{n=0}^{\infty}\psi_n(x,y), \quad \theta = \sum_{n=0}^{\infty}\theta_n(x,y)$ (33)

in equation (32), we arrive at

$$\sum_{n=0}^{\infty}\theta_n(x,y) = C_1 y + C_2 + L_1^{-1}\left[Pe\delta\frac{\partial\left(\sum_{n=0}^{\infty}\psi_n(x,y),\ \sum_{n=0}^{\infty}\theta_n(x,y)\right)}{\partial(y,x)} -\right.$$
$$Br\left(1+\frac{\alpha}{2}\delta\left(\frac{\partial\left(\sum_{n=0}^{\infty}\psi_n(x,y)\right)}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\left(\sum_{n=0}^{\infty}\psi_n(x,y)\right)}{\partial x}\frac{\partial}{\partial y}\right)\right)$$
$$\left(4\delta^2\left(\frac{\partial^2\left(\sum_{n=0}^{\infty}\psi_n(x,y)\right)}{\partial x\partial y}\right)^2+\left(\frac{\partial^2\left(\sum_{n=0}^{\infty}\psi_n(x,y)\right)}{\partial y^2}-\delta^2\frac{\partial^2\left(\sum_{n=0}^{\infty}\psi_n(x,y)\right)}{\partial x^2}\right)^2\right)$$
$$\left.-\delta^2\frac{\partial^2\left(\sum_{n=0}^{\infty}\theta_n(x,y)\right)}{\partial x^2}\right] \quad (34)$$

subject to the boundary conditions on temperature becomes

$$\sum_{n=0}^{\infty}\theta_n(x,y) = 1 \quad at \quad y=f \quad and$$

$$\frac{\partial\left(\sum_{n=0}^{\infty}\theta_n(x,y)\right)}{\partial y} = 0 \quad at \quad y=0. \tag{35}$$

Equation both sides of Equation-34 and 35, we obtain

$$\theta_o = C_1 y + C_2 \tag{36}$$

corresponding boundary conditions are

$$\theta_o = 1 \quad at \quad y=f \quad and$$

$$\frac{\partial\theta_o}{\partial y} = 0 \quad at \quad y=0. \tag{37}$$

Now the expressions for θ_1 and θ_2 becomes

$$\theta_1 = L_1^{-1}\left[Pe\delta\frac{\partial(\psi_o,\theta_o)}{\partial(y,x)} - Br\left(1+\frac{\alpha}{2}\delta\left(\frac{\partial\psi_o}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_o}{\partial x}\frac{\partial}{\partial y}\right)\right)\left(4\delta^2\left(\frac{\partial^2\psi_o}{\partial x\partial y}\right)^2+\left(\frac{\partial^2\psi_o}{\partial y^2}-\delta^2\frac{\partial^2\psi_o}{\partial x^2}\right)^2\right)-\delta^2\frac{\partial^2\theta_o}{\partial x^2}\right] \tag{38}$$

with boundary conditions

$$\theta_1 = 0 \quad at \quad y=f \quad and \quad \frac{\partial\theta_1}{\partial y}=0 \quad at \quad y=0. \tag{39}$$

$$\theta_2 = L_1^{-1}\Bigg[Pe\delta\frac{\partial(\psi_1,\theta_o)}{\partial(y,x)}+Pe\delta\frac{\partial(\psi_o,\theta_1)}{\partial(y,x)} - 2Br\left(1+\frac{\alpha}{2}\delta\left(\frac{\partial\psi_o}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_o}{\partial x}\frac{\partial}{\partial y}\right)\right)$$

and

$$\left(\begin{array}{l}4\delta^2\frac{\partial^2\psi_o}{\partial x\partial y}\frac{\partial^2\psi_1}{\partial x\partial y}+\frac{\partial^2\psi_o}{\partial y^2}\frac{\partial^2\psi_1}{\partial y^2}+\delta^4\frac{\partial^2\psi_o}{\partial x^2}\frac{\partial^2\psi_1}{\partial x^2}\\ -\delta^2\frac{\partial^2\psi_o}{\partial x^2}\frac{\partial^2\psi_1}{\partial y^2}-\delta^2\frac{\partial^2\psi_1}{\partial x^2}\frac{\partial^2\psi_o}{\partial y^2}\end{array}\right)$$

$$-Br\left(1+\frac{\alpha}{2}\delta\left(\frac{\partial\psi_1}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_1}{\partial x}\frac{\partial}{\partial y}\right)\right)\left(\begin{array}{l}4\delta^2\left(\frac{\partial^2\psi_o}{\partial x\partial y}\right)^2\\ +\left(\frac{\partial^2\psi_o}{\partial y^2}-\delta^2\frac{\partial^2\psi_o}{\partial x^2}\right)^2\end{array}\right)-\delta^2\frac{\partial^2\theta_1}{\partial x^2}\Bigg], \tag{40}$$

subject to boundary conditions

$$\theta_2 = 0 \quad at \quad y=f \quad and \quad \frac{\partial\theta_2}{\partial y}=0 \quad at \quad y=0. \tag{41}$$

The solution of (36) by making use of boundary conditions from (37) becomes $\theta_o = 1$. (42)

The solution of (38) along with the boundary conditions from (39) gives

$$\begin{aligned}\theta_1 = -\frac{3Br(\eta^2-1)}{17920f^4}\Big[& -8\alpha\delta f'\{42(2\eta^4-3\eta^2-3)\\ & +3\delta^2f'^2(75\eta^6-177\eta^4+173\eta^2-247)\\ & +2\delta^4f'^4(56\eta^8-124\eta^6+86\eta^4-19\eta^2-19)\}\\ & +2\alpha\delta^3ff'f''\{21(15\eta^6-33\eta^4+37\eta^2-83)\\ & +\delta^2f'^2(616\eta^8-1499\eta^6+1189\eta^4-281\eta^2-281)\}\\ & +4\delta^4f^4f''^2(15\eta^6-41\eta^4+29\eta^2+29)\\ & +\delta^2f^3f''\{224(2\eta^4-3\eta^2-3)\\ & -32\delta^2f'^2(15\eta^6-27\eta^4+8\eta^2+8)\\ & +\alpha\delta^3f^{(3)}(28\eta^8-107\eta^6+145\eta^4-65\eta^2-65)\}\\ & +f^2\{32\delta^4f'^4(30\eta^6-26\eta^4+9\eta^2+9)\\ & -3\alpha\delta^5f'f''^2(84\eta^8-261\eta^6+271\eta^4-79\eta^2-79)\\ & +2(560(\eta^2+1)\\ & +3\alpha\delta^3f^{(3)}(15\eta^6-41\eta^4+29\eta^2+29))\\ & +2\delta^2f'^2\{1120(\eta^4-\eta^2+2)-\alpha\delta^3f^{(3)}\\ & (56\eta^8-169\eta^6+167\eta^4-43\eta^2-43)\}\}\Big].\end{aligned} \tag{43}$$

The solution of (40) up to second order in δ subject to boundary conditions from (41) gives

$$\begin{aligned}\theta_2 = & \frac{3\delta Brf.(\eta^2-1)}{8960f^2}\{2\,\mathrm{Re}(9\eta^6-47\eta^4+19\eta^2+19)\\ & +Pe(15\eta^6-13\eta^4-83\eta^2+337)\}\\ & -\frac{3\delta^2Br.(\eta^2-1)}{179200f^4}\{2f'^2(\alpha(14Pe(8\eta^8-37\eta^6+23\eta^4\\ & +113\eta^2-427)+\mathrm{Re}(196\eta^8-1199\eta^6+1669\eta^4\\ & -461\eta^2-461))-1120f^2(23\eta^4-\eta^2-16))\\ & -ff''(\alpha(4\,\mathrm{Re}(20\eta^8-115\eta^6+161\eta^4-49\eta^2-49)\\ & +7Pe(8\eta^8-37\eta^6+23\eta^4+113\eta^2-427))\\ & -2240f^2(5\eta^4-\eta^2-6))\}+ higher\ order\ of\ \delta.\end{aligned} \tag{44}$$

It is observed that the three orders of θ from ADM and RPM are different from each others.

Wall shear stress

Wall shear stress for second grade fluid in dimensionless form is obtained from the component of Cauchy shear stress as follows:

$$\tau_W=\left\{1+\alpha\left(\delta u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right\}\left(\frac{\partial u}{\partial y}+\delta\frac{\partial v}{\partial x}\right)+2\alpha\frac{\partial v}{\partial y}\left(\delta\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right). \quad (45)$$

Wall shear stress up to second order in δ is obtained by making use of velocity components defined in Equation-5 as follows

$$\tau_W=\frac{3}{f^2}\Big[\ \frac{1}{2}+\frac{\delta f'}{70f^2}\left(2\mathrm{Re}f^2-105\alpha\right)-$$

$$\delta^2\Big\{\ \frac{\mathrm{Re}\alpha}{70f^2}\left(16f'^2-ff''\right)+\frac{\mathrm{Re}^2}{80850}\left(79f'^2-40ff''\right)+\frac{1}{10}\left(13f'^2-2ff''\right)\Big\}\ \Big].. \quad (46)$$

It is found that the wall shear stress by both the method up to second order in δ is same. The points of separation and reattachment are given by setting $\tau_w=0$, the resulting equation is quadratic in Re and the solution in terms of Re is

$$\mathrm{Re}=\frac{7}{2\delta f^2\left(40ff''-79f'^2\right)}\Big[\ 165\left(\alpha\delta\left(16f'^2-ff''\right)-2f^2f'\right)$$

$$\pm\sqrt{165}\sqrt{\begin{array}{l}\{\ 165\left(2f^2f'+\alpha\delta\left(ff''-16f'^2\right)\right)^2\\ -4f^2\left(40ff''-79f'^2\right)\left(2\delta^2f^2f''-15\alpha\delta f'+4f^2\left(5-13\delta^2f'^2\right)\right)\ \}\end{array}}\ \Big]. \quad (47)$$

From the expression (47), we have to find the critical Reynolds number at which the separation and reattachment points occur.

Graphical Discussion

In this section solutions are presented graphically for stream lines, wall shear stress, zero wall shear stress and temperature distribution by ADM and RPM. Solutions are analyzed numerically through graphs for second grade parameter α, height of stenosis ε, Reynolds number (Re), Brinkman number (Br) and Peclet number (Pe).

Figure-2 depict the behavior of stream lines by ADM and RPM for $\mathrm{Re}=12,\ \varepsilon=0.2,\ \delta=0.1,\ \alpha=0.04$. In these figures $x-axis$ lies in the horizontal direction and $y-axis$ is perpendicular to it. The zeroth order solution for stream lines by ADM and RPM are presented in Figures-2(i) and 2(v) respectively; it corresponds to the flow with vanishing wall slopes and reduces to the flow between parallel plates for $\varepsilon=0$. It is observed that the stream lines are relatively straight in the center of the channel. First order solution presented in figures 2(ii) and 2(vi) by ADM and RPM, it is found that first order solution induces the clockwise and counterclockwise rotational motion in the converging and diverging regions, which indicates the separation point in the converging region and reattachment point in the diverging region. Figures-2(iii) and 2(vii) shows the stream lines for second order solution by ADM and RPM, it is found that the rotational motion in both converging and diverging sections predicts the separation and reattachment points. Stream lines up to second order are presented in figures 2(iv) and 2(viii) by ADM and RPM. It is observed that the stream lines becomes relatively straight in the center of the channel and similar to [11] by setting $\alpha=0$. Since the wall shear stress up to second order in δ for both the methods is same, the graphical representation for wall shear stress and points of separation and reattachment by both the methods would be similar. The wall shear stress for various values of Re is shown in Figure-3 for fixed values of $\varepsilon=0.2,\ \delta=0.1,\ \alpha=0.04$. It is observed that an increase in Re, wall shear stress increases near the throat of stenoised region and becomes adverse in the converging and diverging section of the channel. The negative shearing in converging and diverging sections of channel indicates that there is point of separation in the upstream region and reattachment point in the downstream region of the channel. It is found that wall shear stress holds for both small and large Re. It is also observed that the magnitude of adverse wall shear stress in the diverging part is smaller as compared with the converging part. In figure 4 effect of second grade parameter $\alpha=0,\ 0.04,\ 0.08$, is shown on wall shear stress, τ_ω, other parameters are chosen to be $\mathrm{Re}=38,\ \varepsilon=0.7,\ \delta=1/7$. It is observed that for $\alpha=0$, the present result corresponds to viscous fluid and as the second grade parameter increases wall shear stress increases near the throat and becomes negative in converging and diverging sections due to adverse flow. It is noted that the effect of Re and second grade parameter on wall shear stress have similar adverse behavior. It is also noted that the magnitude of adverse flow in the diverging region is smaller as compared to converging region. In Figure-5 effect of ε on wall shear stress is presented. The straight line indicates that there is no stenosis and the flow is Poiseuille flow. It is found that by the increase in ε wall shear stress increases near the throat and becomes negative in the converging and diverging sections of the channel, which is the prediction for the points of separation and reattachment. The separation point was considered to be the point nearest the throat where adverse flow along the wall of channel is observed. The point farthest downstream from the throat where back flow occurs is defined as reattachment point. Figure 6 presents the distribution for the point of separation in converging section of the channel for different values of ε along with fixed values of δ and α. The separation point lies to the right of minimum point; actually the purpose for zero wall shear stress is to find the critical Reynolds number where separation occurs. It is observed that the critical Re decreases as the ε increases. The theory that the critical Reynolds number decreases with the increase in height of stenosis is verified. Figure-7 predicts the separation point for different values of second grade parameter (α) in the converging region for fixed values of δ and ε. It is observed that with the increase in α critical Reynolds number (Re) decreases. It is observed that the critical Re have same behavior for negative values of α. In Figure-8 zero wall shear stress is plotted for ε having fixed values of α and δ in diverging section of the channel. The aim of investigation is to determine the critical value of Re at which reattachment occurred in the diverging region of the channel. As the critical Re reached the reattachment occurred in the diverging region of the channel and separation point occurred in the upstream region of the channel. It is observed form figure 8 that as ε increases critical Re decreases. In figure 9 analysis of zero wall shear stress is presented for various values of α along with fixed values of ε and δ. It is observed that critical Re decreases as α increases in the diverging region of the channel. It is noted that the reattachment point lies to the left of minimum point and found the similar behavior as in figure 8. Now the numerical results are carried out to study the behavior of temperature distribution graphically for α, Br, and Pe by ADM and RPM. In figures 10 and 11, behavior of Newtonian $\alpha = 0$ and non-Newtonian $\alpha \neq 0$ effects are observed for the distribution of temperature by ADM and RPM respectively. It is noted that by the increase in α temperature increases over the stenoised region with fixed values of the remaining parameters and becomes negative in the converging and diverging regions with small amplitude. It is found that the maximum value of temperature occurs at the middle of the stenoised region. The adverse temperature in these regions causes back flow as observed earlier. In Figures-12 and 13 the effect of Br number is shown over the distribution of temperature respectively by ADM and RPM for fixed Pe and Re. It is found that with the increase in Br temperature increases over the stenosis. Figures 14 and 15 presents the effect of Peclet number, Pe, on temperature by keeping other parameters fixed. It is found that with the increase in Pe number temperature increases by two methods which firmly ensure that the whole region is dominated by convection. The magnitude of the adverse temperature in the diverging regions is smaller as compared to the converging regions. The adverse temperature in these sections causes back flow.

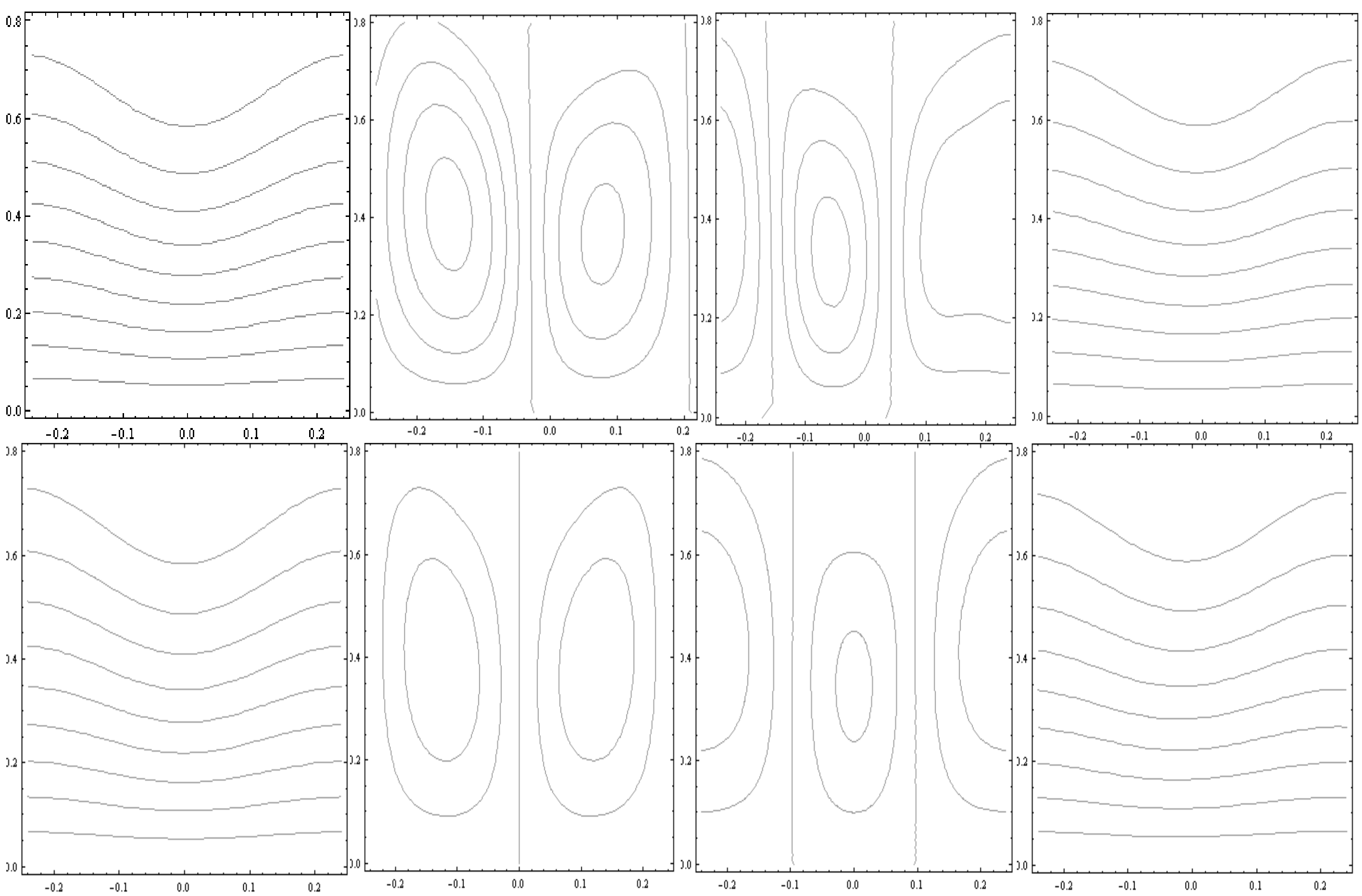

Figure-2
Stream lines

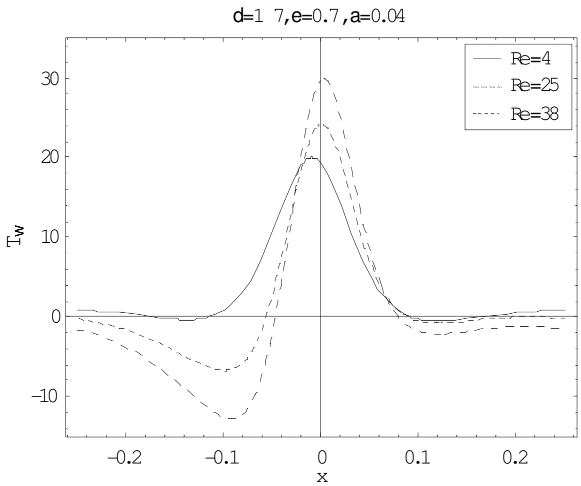

Figure-3
Effect of Re on wall shear stress

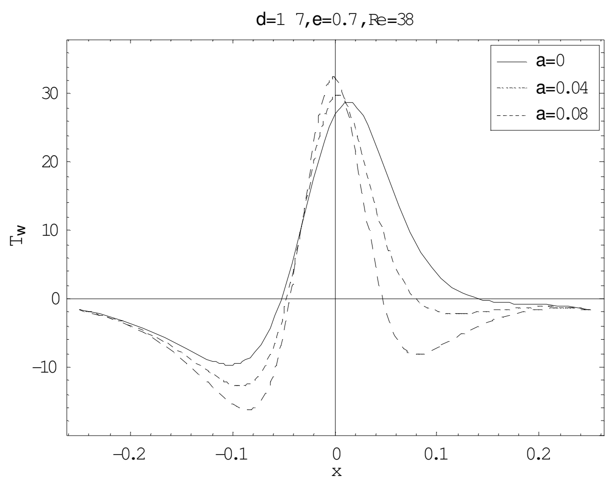

Figure-4
Effect of α on wall shear stress

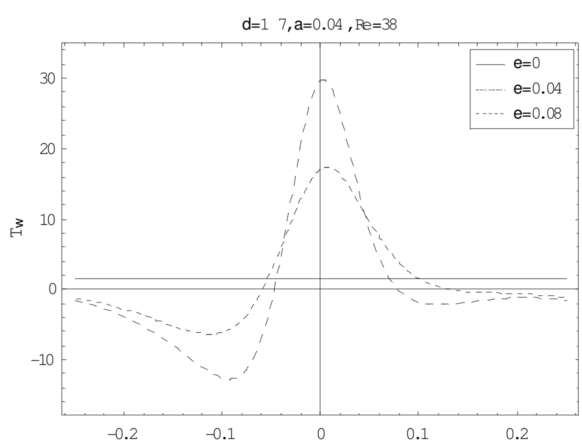

Figure-5
Effect of ε on wall shear stress

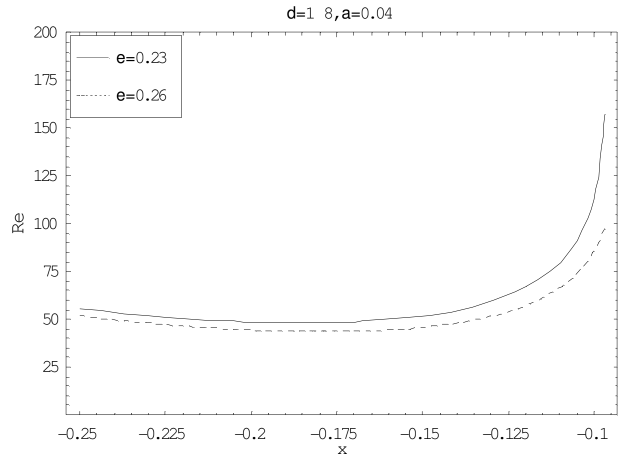

Figure-6
Separation point for ε in converging region

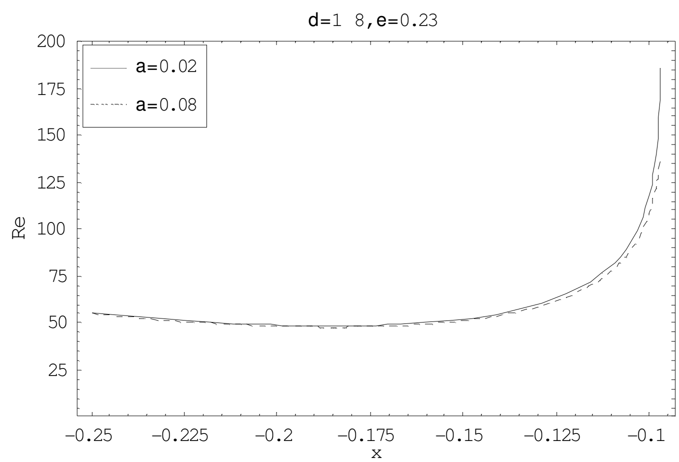

Figure-7
Separation point for α in the converging region

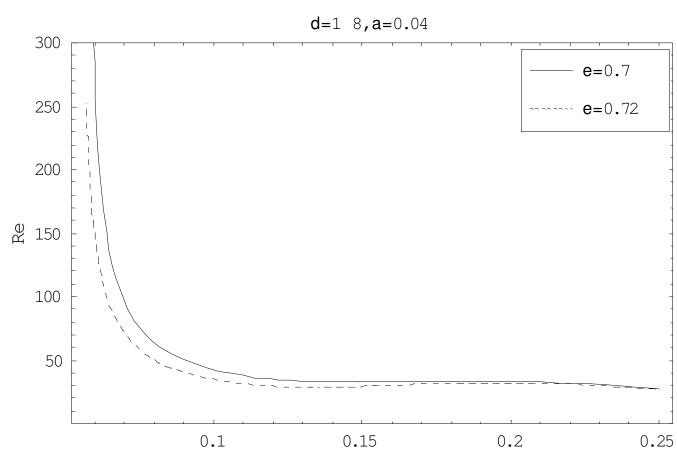

Figure-8
Reattachment point for ε in the diverging region

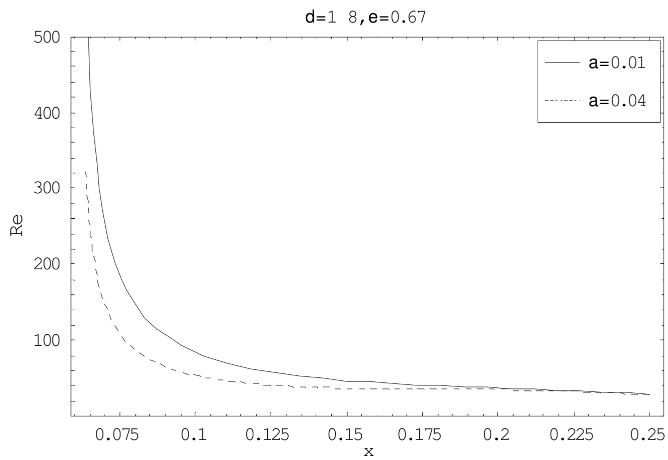

Figure-9
Reattachment point for α in the diverging region

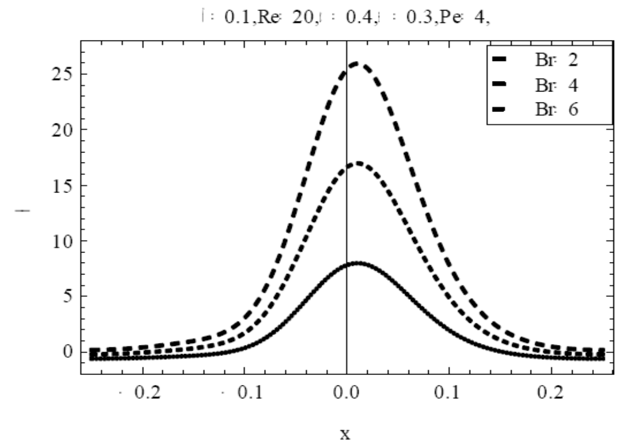

Figure-12
Temperature distribution for Br by ADM

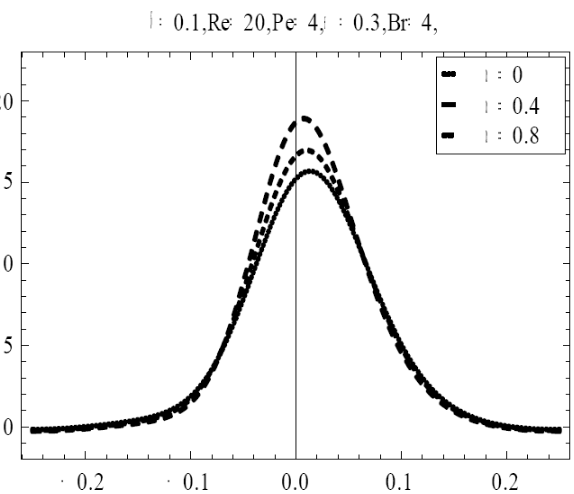

Figure-10
Temperature distribution for α by ADM

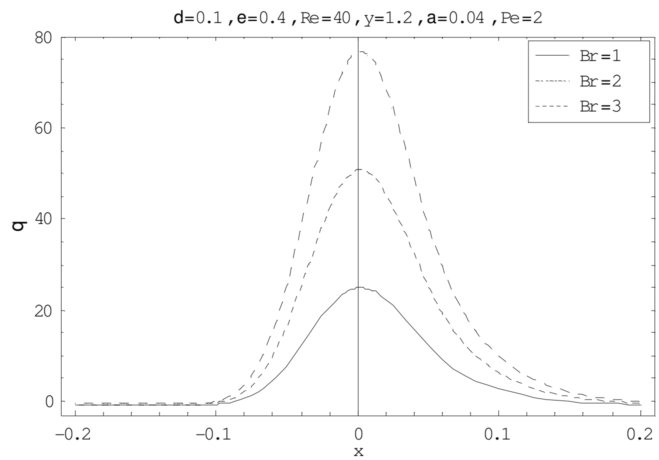

Figure-13
Temperature distribution for Br by RPM

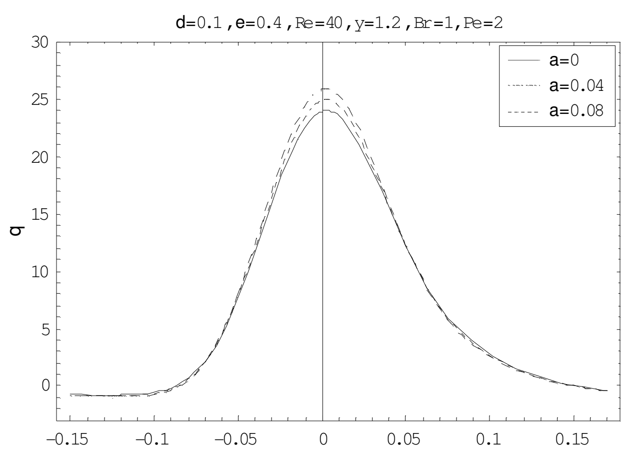

Figure-11
Temperature distribution for α by RPM

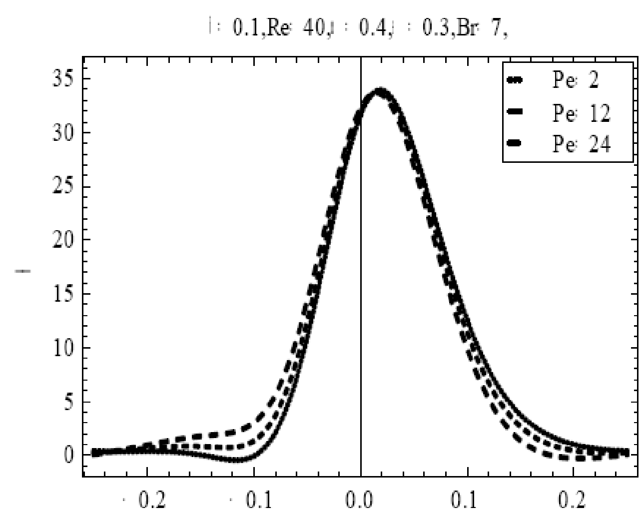

Figure-14
Temperature distribution for Pe by ADM

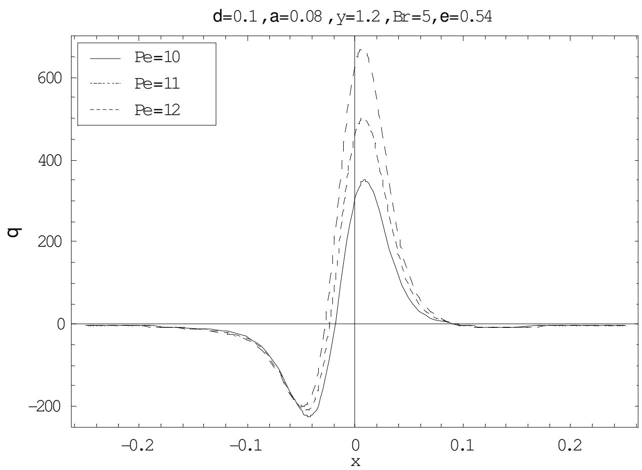

Figure-15
Temperature distribution for Pe by RPM

Conclusion

In present article, consideration has been given to second grade steady state flow of blood through the channel of infinite length with heat transfer having stenosis of length $l_o/2$. Non-linear equations are solved by ADM and regular perturbation method (RPM). The results thus obtained are discussed graphically in terms of stream lines, wall shear stress, separation and reattachment points and temperature distribution. It is noted that by setting $\alpha = 0$, the present model reduces to viscous case similar to previously reported results[6-10]. Furthermore, the general pattern of streamlines is same as discussed by previous researchers[8-10]. Wall shear stress is similar to J.H. et. al.[7] & Lee J.S. et. al.[8] and separation and reattachment points are similar to Lee J.S. et. al.[8]. From the present investigation the following conclusions are made: Increase in Re increases wall shear stress. Increase in ε increases wall shear stress and temperature. Critical Re decreases with an increase in ε. Increase in α leads to increases in temperature and wall shear stress. Temperature increases with an increase in Br and Pe. The critical Re decreases with an increase in α in the converging and diverging region. As a comparison ADM gives better results than RPM and easy to compute.

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